PS-I, UNIT - I

Series parameters of Overhead Transmission Lines

Learning objectives:

- List out different types of conductors
- Name different types of transmission lines and its parameters
- Derive the expression for inductance for single and three phase systems
- Describe the transposition of transmission lines
- Define G.M.D and G.M.R

Syllabus:

Structure of power systems, classification of transmission systems, Types of conductors - Transmission line parameters- calculation of resistance for solid conductors - Calculation of inductance for single phase and three phase, single and double circuit lines, concept of GMR & GMD, symmetrical and asymmetrical conductor configuration with and without transposition, Numerical Problems.

Learning outcomes:

Students will be able to

- explain about types of conductors and transmission line parameters.
- obtain the expression for inductance for single and three phase systems
- determine G.M.D and G.M.R of transmission lines.

Learning Material

Series parameters of Overhead Transmission Lines

Introduction of Modern Power System

Modern electric power systems have three separate components - generation, transmission and distribution. Electric power is generated at the power generating stations by synchronous alternators that are usually driven either by steam or hydro turbines. Most of the power generation takes place at generating stations that may contain more than one such alternator-turbine combination. Depending upon the type of fuel used, the generating stations are categorized as thermal, hydro, nuclear etc. Many of these generating stations are remotely located. Hence the electric power generated at any such station has to be transmitted over a long distance to load centers that are usually cities or towns. This requires **power transmission**. In fact power transmission towers and transmission lines are very common sights in rural areas.

Modern day power systems are complicated networks with hundreds of generating stations and load centers being interconnected through power transmission lines. Electric power is generated at a frequency of 50 Hz or 60 Hz.

In an interconnected ac power system, the rated generation frequency of all units must be the same. In India the frequency is 50 Hz.

Basic Structure of a Power System

The basic structure of a power system is shown in Fig. 1.1.

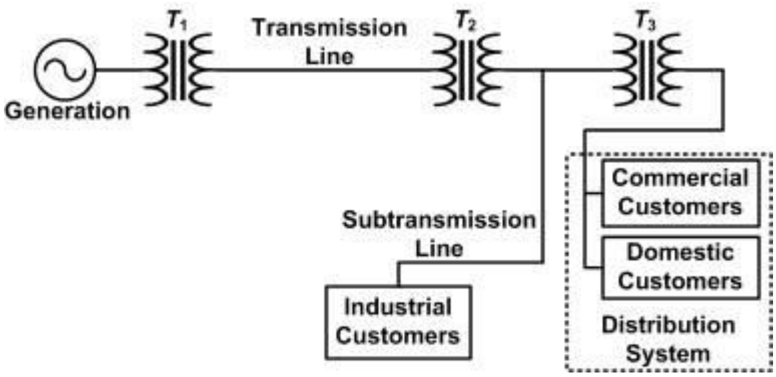
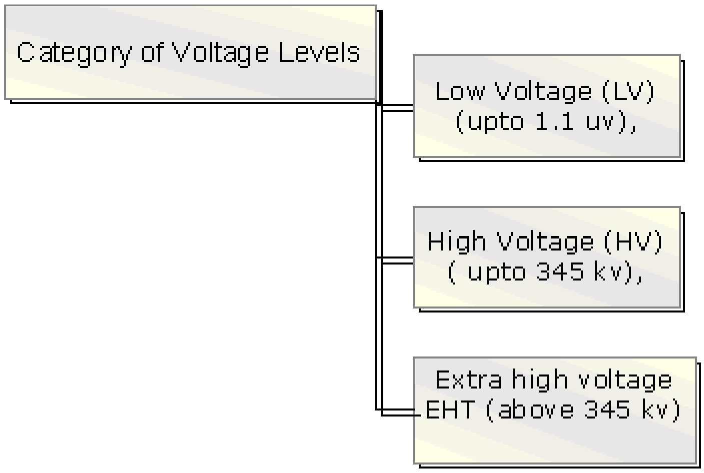


Fig. 1.1 A typical power system.

It contains a generating plant, a transmission system, a sub transmission system and a distribution system. These subsystems are interconnected through transformers T_1 , T_2 and T_3 . Let us consider some typical voltage levels to understand the functioning of the power system. The electric power is generated at a thermal plant with a typical voltage of 22 kV (voltage levels are usually specified line-to-line). This is boosted up to levels like 400 kV through transformer T_1 for power transmission. Transformer T_2 steps this voltage down to 66 kV to supply power through the sub transmission line to industrial loads that require bulk power at a higher voltage. Most of the major industrial customers have their own transformers to step down the 66 kV supply to their desired levels. The motivation for these voltage changes is to minimize transmission line cost for a given power level. Distribution systems are designed to operate for much lower power levels and are supplied with medium level voltages.



The power distribution network starts with transformer T_3, which steps down the voltage from 66 kV to 11 kV. The distribution system contains loads that are either commercial type (like office buildings, huge apartment complexes, hotels etc) or residential (domestic) type. Usually the commercial customers are supplied power at a voltage level of 11 kV whereas the domestic consumers get power supply at 400-440 V. Note that the above figures are given for line-to-line voltages. Since domestic customers get single-phase supplies, they usually receive 230-250 V at their inlet points. While a domestic customer with low power consumption gets a single-phase supply, both industrial and commercial consumers get three-phase supplies not only because their consumption is high but also because many of them use three-phase motors. For example, the use of induction motor is very common amongst industrial customers who run pumps, compressors, rolling mills etc.

The main components of a power system are generators, transformers and transmission lines.

In this chapter we shall discuss the models of transmission lines that will be used subsequently in power system studies.

Materials commonly used in conductors are aluminium, copper, and steel. Galvanized steel wires are combined with aluminium in the most common type of overhead conductor -- Aluminium Conductor Steel Reinforced (ACSR). The use of copper is uncommon in modern transmission lines since it weighs and usually costs considerably more than aluminium conductor of the same resistance.

Types of Aluminium Conductors:

(i) All Aluminium Conducts (AAC)
(ii) All Alloy Aluminium Conductors (AAAC), and
(iii) Aluminium Conductors Steel Reinforced (ACSR)

ACSR conductors are used in Transmission and Distribution system to carry the generated electrical energy from generating station to end user. The Electrical energy is normally generated at the power stations far away from the urban areas where the consumers are located. There is a large network of conductors between the generating stations and the consumer. The network is called the Transmission and Distribution system. The Transmission system is to deliver bulk power from power stations to the load centres and large industrial consumers beyond the economical service range of the regular primary distribution lines where as distribution system is to deliver power from power sector or substations to the various consumers. This transmission and distribution system can employ either overhead system or underground system. Transmission of power, overhead system mainly due to low cost and some other advantages ACSR generally used or transmission line and AAC and AAAC conductors for distribution of power carry out mostly the high voltage transmission. For transmission and distribution of electric power the conductor material used must have the following characteristics:

i) High conducting i.e. low specific resistance
ii) High tensile strength in order to withstand mechanical stress
iii) Low specific gravity in order to give low weight per unit volume
iv) Low cost in order to be used over long distance
v) Should not be brittle

Copper, Aluminium, Steel and Steel cored aluminium, galvanised steel conductors are generally employed for this purpose and preferably stranded in order to increase the flexibility (Solid wires, except of smaller sizes, are difficult to handle and when employed for long spans tend to crystallize at the points of support because of the swinging in winds.

Stranded conductors usually have a central wire around which there are successive layers of 6, 12, 18, 24 wires. For 19 layers, the total number of individual wire is 3n (n+1). If the diameter of each strand is torn diameter of the stranded conductors will be (2n+1) d. In the process of manufacture adjacent layers are spiraled in opposite direction, so that the layers are bound together. This method of construction is called as Concentric Lay, out of above mentioned conducting materials, Aluminium is widely used due to its cheapness and many others comparative advantages over other conducting materials. However, owing to the fact that the line or co-efficient of expansion of aluminium is 104 times that of copper, the sag is greater in aluminium wire, therefore, steel cored Aluminium (ACSR) wire is used to compensate this property of Aluminium. The steel conductors used are galvanised in order to prevent rusting and electrolytic corrosion. The AAC/AAAC/ACSR conductors for high voltage transmission have first replaced the bore copper conductors where copper is scarce and costly on the other hand EC grade Aluminium is easily available in India and as far as the electric properties are concerned, aluminium is equally good being lighter in weight and for same sage span length of the transmission could be increased in comparison to copper. Keeping in view the simple technology involved AAC/ACSR/AAAC conductors up to 19 strand have been reserved for exclusive production in small scale sector. However, Aluminium conductors up to 61 strand can be manufactured. Different types of aluminium conductors manufactured are:

i) All Aluminium stranded conductors (AAC)
ii) Aluminium conductors, aluminized steel reinforced
iii) Aluminium conductors galvanised steel reinforced (ACSR)
iv) All Aluminium Alloy stranded Conductors (AAAC)
v) Aluminium conductors galvanised steel reinforced for extra high voltage (400 kV or above) (ACSR)

Series Parameters of Transmission Lines

Overhead transmission lines and transmission towers are a common sight in rural India. The transmission towers are usually made of steel and are solidly erected with a concrete base. The three-phase conductors are supported by the towers through insulators. The conductors are usually made of aluminium or its alloys. Aluminium is preferred over copper as an aluminium conductor is lighter in weight and cheaper in cost than copper conductor of the same resistance.

The conductors are not straight wires but strands of wire twisted together to form a single conductor to give it higher tensile strength. One of the most common conductors is aluminium conductor, steel reinforced (ACSR). The cross sectional view of such a conductor is shown in Fig. 1.2. The central core is formed with strands of steel while two layers of aluminium strands are put in the outer layer.

The other type of conductors that are in use are all aluminium conductor (AAC), all aluminium alloy conductor (AAAC), aluminium conductor, alloy reinforced (ACAR).

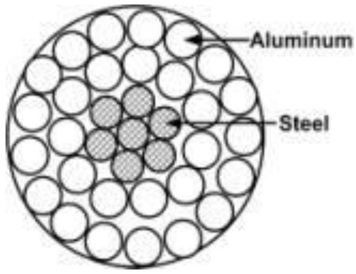


Fig. 1.2 Cross sectional view of an ACSR conductor.

Line Resistance

It is very well known that the dc resistance of a wire is given by

$$R_{dc} = \frac{\rho l}{A} \tag{1.1}$$

Where ρ is the resistivity of the wire in **Ω** - m, l is the length in meter and A is the cross sectional area in m^2. Unfortunately however the resistance of an overhead conductor is not the same as that given by the above expression. When alternating current flows through a conductor, the current density is not uniform over the entire cross section but is somewhat higher at the surface. This is called the **skin effect** and this makes the ac resistance a little more than the dc resistance. Moreover in a stranded conductor, the length of each strand is more that the length of the composite conductor. This also increases the value of the resistance from that calculated in (1.1).

Finally the temperature also affects the resistivity of conductors. However the temperature rise in metallic conductors is almost linear in the practical range of operation and is given by

$$\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1} \tag{1.2}$$

where R_1 and R_2 are resistances at temperatures t_1 and t_2 respectively and T is a constant that depends on the conductor material and its conductivity. Since the resistance of a conductor cannot be determined accurately, it is best to determine it from the data supplied by the manufacturer

Inductance of a Straight Conductor

From the knowledge of high school physics we know that a current carrying conductor produces a magnetic field around it. The magnetic flux lines are concentric circles with their direction specified by **Maxwell's right hand thumb rule** (i.e., if the thumb of the right hand points towards the flow of current then the fingers of the fisted hand point towards the flux lines). The sinusoidal variation in the current produces a sinusoidal variation in the flux. The relation between the inductance, flux linkage and the phasor current is then expressed as

$$L = \frac{\lambda}{I} \tag{1.3}$$

Where L is the inductance in Henry, λ is the flux linkage in Weber-turns and I is the phasor current in Ampere.

A. Internal Inductance

Consider a straight round (cylindrical) conductor, the cross-section of which is shown in Fig. 1.3. The conductor has a radius of r and carries a current I. Ampere's law states that the magnetomotive force (mmf) in ampere-turns around a closed path is equal to the net current in amperes enclosed by the path. We then get the following expression

$$mmf = \oint H \cdot ds = I \tag{1.4}$$

where H is the magnetic field intensity in At/m, s is the distance along the path in meter and I is the current in ampere.

Let us denote the field intensity at a distance x from the center of the conductor by H_x. It is to be noted that H_x is constant at all points that are at a distance x from the center of the conductor. Therefore H_x is constant over the concentric circular path with a radius of x and is tangent to it. Denoting the current enclosed by I_x we can then write

$$\oint H_x \cdot dx = I_x \Rightarrow H_x = \frac{I_x}{2\pi x} \tag{1.5}$$

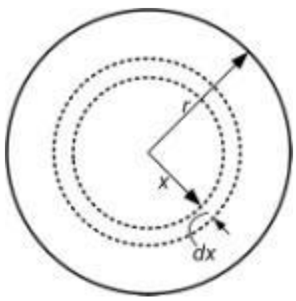


Fig. 1.3 Cross section of a round conductor.

If we now assume that the current density is uniform over the entire conductor, we can write

$$\frac{I}{\pi r^2} = \frac{I_x}{\pi x^2} \Rightarrow I_x = \frac{\pi x^2}{\pi r^2} I \tag{1.6}$$

Substituting (1.6) in (1.5) we get

$$H_x = \frac{I}{2\pi r^2} x \tag{1.7}$$

Assuming a relative permeability of 1, the flux density at a distance of *x* from the center of the conductor is given by

$$B_x = \mu_0 H_x = \frac{\mu_0 I}{2\pi r^2} x \tag{1.8}$$

where μ_0 is the permeability of the free space and is given by $4\pi \times 10^{-7}$ H/m.

The flux inside (or outside) the conductor is in the **circumferential direction** . The two directions that are perpendicular to the flux are radial and axial . Let us consider an elementary area that has a dimension of *dx*m along the radial direction and 1 m along the axial direction. Therefore the area perpendicular to the flux at all angular positions is dx X 1 m^2 . Let the flux along the circular strip be denoted by $d\varphi_x$ and this is given by

$$d\phi_x = B_x dx \times 1 = \frac{\mu_0 I}{2\pi r^2} x\, dx \tag{1.9}$$

Note that the entire conductor cross section does not enclose the above flux. The ratio of the cross sectional area inside the circle of radius *x* to the total cross section of the conductor can be thought about as fractional turn that links the flux $d\varphi_x$. Therefore the flux linkage is

$$d\lambda_x = \frac{\pi x^2}{\pi r^2} d\phi_x = \frac{\mu_0 I}{2\pi r^4} x^3\, dx \tag{1.10}$$

Integrating (1.10) over the range of *x* , i.e., from 0 to *r* , we get the internal flux linkage as

$$\lambda_{int} = \int_0^r \frac{\mu_0 I}{2\pi r^4} x^3 dx = \frac{\mu_0 I}{8\pi} = \frac{I}{2} \times 10^{-7} \text{ Wbt/m} \tag{1.11}$$

Then from (1.3) we get the internal inductance per unit length as

$$L_{int} = \frac{1}{2} \times 10^{-7} \text{ H/m} \tag{1.12}$$

For $\mu \neq 1$,

$$L_{int} = \frac{1}{2} \times 10^{-7} \mu_r \text{ H/m}$$

It is interesting to note that the internal inductance is independent of the conductor radius.

B. External Inductance

Let us consider an isolated straight conductor as shown in Fig. 1.4. The conductor carries a current *I* . Assume that the tubular element at a distance *x* from the center of the conductor has a field intensity H_x . Since the circle with a radius of *x* encloses the entire current, the mmf around the element is given by

$$2\pi x H_x = I \tag{1.13}$$

and hence the flux density at a radius *x* becomes

$$B_x = \frac{\mu_0 I}{2\pi x} \tag{1.14}$$

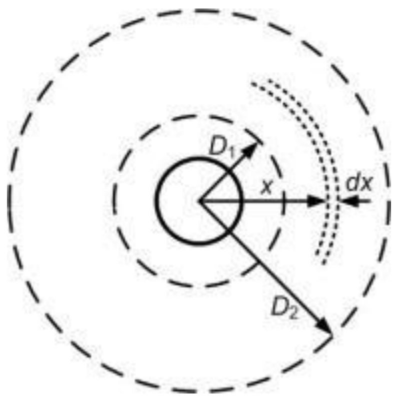


Fig 1.4 A Conductor with two external points

The entire current *I* is linked by the flux at any point outside the conductor. Since the distance *x* is greater than the radius of the conductor, the flux linkage $d\lambda_x$ is equal to the flux $d\varphi_x$. Therefore for 1 m length of the conductor we get

$$d\lambda_x = d\phi_x = B_x dx \cdot 1 = \frac{\mu_0 I}{2\pi x} dx \tag{1.15}$$

The external flux linkage between any two points D_1 and D_2, external to the conductor is

$$\lambda_{ext} = \frac{\mu_0 I}{2\pi} \int_{D_1}^{D_2} \frac{1}{x} dx = 2 \times 10^{-7} I \ln \frac{D_2}{D_1} \text{ Wbt/m} \tag{1.16}$$

From (1.3) we can then write the inductance between any two points outside the conductor as

$$L_{ext} = 2\times10^{-7}\ln\frac{D_2}{D_1} \text{ H/m} \tag{1.17}$$

For μ≠1,

$$L_{ext} = 2\times10^{-7}\ln\frac{D_2}{D_1} \text{ H/m}$$ where μ_r =relative permeability

Inductance of a Single-phase Line

Consider two solid round conductors with radii of r_1 and r_2 as shown in Fig. 1.5. One conductor is the return circuit for the other. This implies that if the current in conductor 1 is I then the current in conductor 2 is $-I$. First let us consider conductor 1. The current flowing in the conductor will set up flux lines. However, the flux beyond a distance $D + r_2$ from the center of the conductor links a net current of zero and therefore does not contribute to the flux linkage of the circuit. Also at a distance less than $D - r_2$ from the center of conductor 1 the current flowing through this conductor links the flux. Moreover since $D >> r_2$ we can make the following approximations

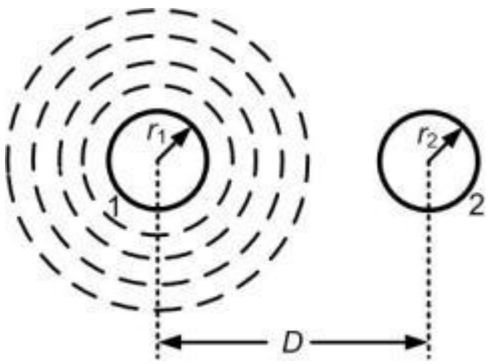


Fig. 1.5 A single-phase line with two conductors.

$$D + r_1 \approx D \text{ and } D - r_1 \approx D$$

Therefore from (1.12) and (1.17) we can specify the inductance of conductor 1 due to internal and external flux as

$$L_1 = \left(\frac{1}{2} + 2\ln\frac{D}{r_1}\right)\times10^{-7} \text{ H/m} \tag{1.18}$$

We can rearrange L_1 given in (1.18) as follows

$$L_1 = 2\times10^{-7}\left(\frac{1}{4}+\ln\frac{D}{r_1}\right) = 2\times10^{-7}\left(\ln e^{1/4}+\ln\frac{D}{r_1}\right) = 2\times10^{-7}\left(\ln\frac{D}{r_1 e^{-1/4}}\right)$$

Substituting $r_1' = r_1 e^{-1/4}$ in the above expression we get

$$L_1 = 2\times10^{-7}\left(\ln\frac{D}{r_1'}\right) \text{ H/m} \tag{1.19}$$

The radius r_1' can be assumed to be that of a fictitious conductor that has no internal flux but with the same inductance as that of a conductor with radius r_1.

In a similar way the inductance due current in the conductor 2 is given by

$$L_2 = 2\times10^{-7}\left(\ln\frac{D}{r_2'}\right) \text{ H/m} \tag{1.20}$$

Therefore the inductance of the complete circuit is

$$\begin{aligned} L = L_1 + L_2 &= 2\times10^{-7}\left(\ln\frac{D}{r_1'}\right) + 2\times10^{-7}\left(\ln\frac{D}{r_2'}\right) \\ &= 2\times10^{-7}\left(\ln\frac{D^2}{r_1' r_2'}\right) = 4\times10^{-7}\left(\ln\frac{D}{\sqrt{r_1' r_2'}}\right) \text{ H/m} \quad \text{H/m} \end{aligned} \tag{1.21}$$

If we assume $r_1 = r_2 = r$, then the total inductance becomes

$$L = 4\times10^{-7}\left(\ln\frac{D}{r'}\right) \text{ H/m} \tag{1.22}$$

where $r' = re^{-1/4}$.

Inductance of Three-Phase Lines with Symmetrical Spacing

Consider the three-phase line shown in Fig. 1.6. Each of the conductors has a radius of r and their centers form an equilateral triangle with a distance D between them. Assuming that the currents are balanced, we have

$$I_a + I_b + I_c = 0 \tag{1.23}$$

Consider a point P external to the conductors. The distance of the point from the phases a, b and c are denoted by D_{pa}, D_{pb} and D_{pc} respectively.

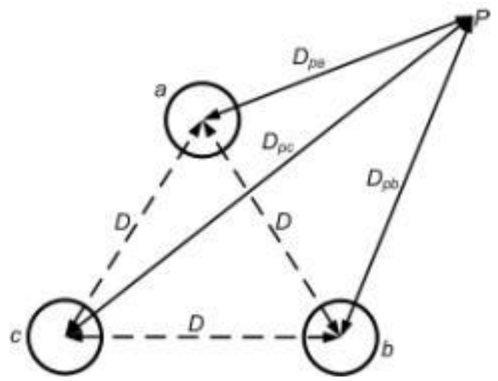


Fig. 1.6 Three-phase symmetrically spaced conductors and an external point P.

Let us assume that the flux linked by the conductor of phase-a due to a current I_a includes the internal flux linkages but excludes the flux linkages beyond the point *P*. Then from (1.18) we get

$$\lambda_{apa} = \left(\frac{1}{2} + 2\ln\frac{D_{pa}}{r}\right)I_a = 2\times10^{-7} I_a \ln\frac{D_{pa}}{r'} \tag{1.24}$$

The flux linkage with the conductor of phase-a due to the current I_b , excluding all flux beyond the point *P* , is given by (1.17) as

$$\lambda_{apb} = 2\times10^{-7} I_b \ln\frac{D_{pb}}{D} \tag{1.25}$$

Similarly the flux due to the current I_c is

$$\lambda_{apc} = 2\times10^{-7} I_c \ln\frac{D_{pc}}{D} \tag{1.26}$$

Therefore the total flux in the phase-a conductor is

$$\lambda_a = \lambda_{apa} + \lambda_{apb} + \lambda_{apc} = 2\times10^{-7}\left(I_a \ln\frac{D_{pa}}{r'} + I_b \ln\frac{D_{pb}}{D} + I_c \ln\frac{D_{pc}}{D}\right)$$

The above expression can be expanded as

$$\lambda_a = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D} + I_c \ln\frac{1}{D} + I_a \ln D_{pa} + I_b \ln D_{pb} + I_c \ln D_{pc}\right) \tag{1.27}$$

From (1.23) we get

$$I_b + I_c = -I_a$$

Substituting the above expression in (1.27) we get

$$\lambda_a = 2\times 10^{-7}\left(I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D} + I_b \ln \frac{D_{pb}}{D_{pa}} + I_c \ln \frac{D_{pc}}{D_{pa}} \right) \tag{1.28}$$

Now if we move the point *P* far away, then we can approximate D_{pa} □ D_{pb} □ D_{pc}.. Therefore their logarithmic ratios will vanish and we can write (1.28) as

$$\lambda_a = 2\times 10^{-7}\left(I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D} \right) = 2\times 10^{-7} I_a \ln \frac{D}{r'} \tag{1.29}$$

Hence the inductance of phase-a is given as

$$L_a = 2\times 10^{-7} \ln \frac{D}{r'} \tag{1.30}$$

Note that due to symmetry, the inductances of phases b and c will be the same as that of phase-a given above, i.e., $L_b = L_c = L_a$.

Inductance of Three-Phase Lines with Asymmetrical Spacing

It is rather difficult to maintain symmetrical spacing as shown in Fig. 1.6 while constructing a transmission line. With asymmetrical spacing between the phases, the voltage drop due to line inductance will be unbalanced even when the line currents are balanced. Consider the three-phase asymmetrically spaced line shown in Fig. 1.7 in which the radius of each conductor is assumed to be r . The distances between the phases are denoted by D_{ab}, D_{bc} and D_{ca}. We then get the following flux linkages for the three phases

$$\lambda_a = 2\times 10^{-7}\left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D_{ab}} + I_c \ln \frac{1}{D_{ca}} \right) \tag{1.31}$$

$$\lambda_b = 2\times 10^{-7}\left(I_b \ln \frac{1}{r'} + I_a \ln \frac{1}{D_{ab}} + I_c \ln \frac{1}{D_{bc}} \right) \tag{1.32}$$

$$\lambda_c = 2\times 10^{-7}\left(I_c \ln \frac{1}{r'} + I_a \ln \frac{1}{D_{ca}} + I_b \ln \frac{1}{D_{bc}} \right) \tag{1.33}$$

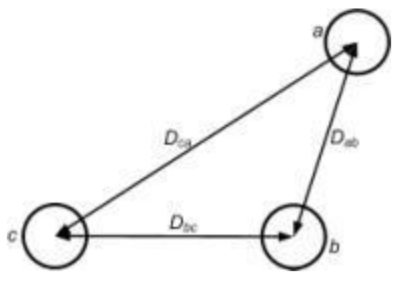


Fig. 1.7 Three-phase asymmetrically spaced line.

Let us define the following operator

$$a = e^{j120^0} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \tag{1.34}$$

Note that for the above operator the following relations hold

$$a^2 = e^{j240^0} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \text{ and } 1 + a + a^2 = 0 \tag{1.35}$$

Let as assume that the current are balanced. We can then write

$$I_b = a^2 I_a \text{ and } I_c = aI_a$$

Substituting the above two expressions in (1.31) to (1.33) we get the inductance of the three phases as

$$L_a = 2\times 10^{-7}\left(\ln\frac{1}{r'} + a^2\ln\frac{1}{D_{ab}} + a\ln\frac{1}{D_{ca}}\right) \tag{1.36}$$

$$L_b = 2\times 10^{-7}\left(\ln\frac{1}{r'} + a\ln\frac{1}{D_{ab}} + a^2\ln\frac{1}{D_{bc}}\right) \tag{1.37}$$

$$L_c = 2\times 10^{-7}\left(\ln\frac{1}{r'} + a^2\ln\frac{1}{D_{ca}} + a\ln\frac{1}{D_{bc}}\right) \tag{1.38}$$

It can be seen that the inductances contain imaginary terms. The imaginary terms will vanish only when $D_{ab}= D_{bc}=D_{ca}$. In that case the inductance will be same as given by (1.30).

Transposition of Lines

The inductances that are given in (1.36) to (1.38) are undesirable as they result in an unbalanced circuit configuration. One way of restoring the balanced nature of the circuit is to exchange the positions of the conductors at regular intervals. This is called transposition of line and is shown in **Fig.1.8.** In this each segment of the line is divided into three equal sub-segments. The conductors of each of the phases a, b and c are exchanged after every sub-segment such that each of them is placed in each of the three positions once in the entire segment. For example, the conductor of the phase-a occupies positions in the sequence 1, 2 and 3 in the three sub-segments while that of the phase-b occupies 2, 3 and 1. The transmission line consists of several such segments.

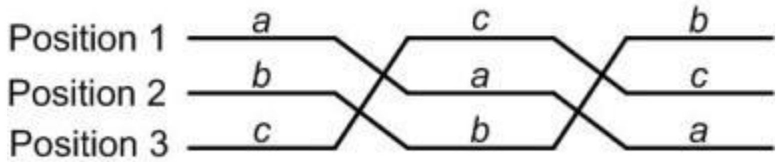


Fig. 1.8 A segment of a transposed line.

In a transposed line, each phase takes all the three positions. The per phase inductance is the average value of the three inductances calculated in (1.36) to (1.38). We therefore have

$$L = \frac{L_a + L_b + L_c}{3} \tag{1.39}$$

This implies

$$L = \frac{2\times 10^{-7}}{3}\left[\ln\frac{3}{r'} + (a + a^2)\left(\ln\frac{1}{D_{ab}} + \ln\frac{1}{D_{bc}} + \ln\frac{1}{D_{bc}}\right)\right]$$

From (1.35) we have $a + a^2 = -1$. Substituting this in the above equation we get

$$L = \frac{2\times 10^{-7}}{3}\left(3\ln\frac{1}{r'} - \ln\frac{1}{D_{ab}} - \ln\frac{1}{D_{bc}} - \frac{1}{D_{ca}}\right) \tag{1.40}$$

The above equation can be simplified as

$$L = 2\times 10^{-7}\left(\ln\frac{1}{r'} - \ln\frac{1}{(D_{ab}D_{bc}D_{ca})^{1/3}}\right) = 2\times 10^{-7}\ln\frac{(D_{ab}D_{bc}D_{ca})^{1/3}}{r'} \tag{1.41}$$

Defining the **geometric mean distance (GMD)** as

$$GMD = \sqrt[3]{D_{ab}D_{bc}D_{ca}} \tag{1.42}$$

equation (1.41) can be rewritten as

$$L = 2\times 10^{-7}\ln\frac{GMD}{r'} \text{ H/m} \tag{1.43}$$

Notice that (1.43) is of the same form as (1.30) for symmetrically spaced conductors. Comparing these two equations we can conclude that *GMD* can be construed as the equivalent conductor spacing. The *GMD* is the cube root of the product of conductor spacings.

Inductance of Composite Conductors

So far we have considered only solid round conductors. However as mentioned at the beginning, stranded conductors are used in practical transmission line. We must therefore modify the equations derived above to accommodate stranded conductors. Consider the two groups of conductors shown in **Fig. 1.9**. Of these two groups conductor x contains n identical strands of radius r_x while conductor y contains m identical strands of radius r_y. Conductor x carries a current I the return path of which is through conductor y. Therefore the current through conductor y is $-I$.

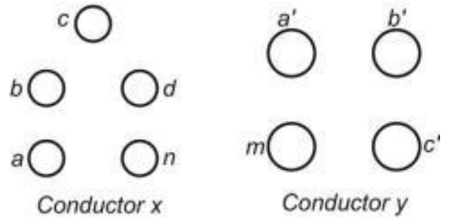


Fig. 1.9 Single-phase line with two composite conductors.

Since the strands in a conductor are identical, the current will be divided equally among the strands. Therefore the current through the strands of conductor *x* is I / n and through the strands of conductor *y* is $-I/m$. The total flux linkage of strand *a* is given by

$$\begin{aligned}\lambda_a &= 2\times10^{-7}\frac{I}{n}\left(\ln\frac{1}{r_x'}+\ln\frac{1}{D_{ab}}+\ln\frac{1}{D_{ac}}+\cdots+\ln\frac{1}{D_{an}}\right)\\ &\quad -2\times10^{-7}\frac{I}{m}\left(\ln\frac{1}{D_{aa'}}+\ln\frac{1}{D_{ab'}}+\ln\frac{1}{D_{ac'}}+\cdots+\ln\frac{1}{D_{am'}}\right)\end{aligned} \tag{1.44}$$

We can write (1.44) as

$$\lambda_a = 2\times10^{-7} I \ln\frac{\sqrt[m]{D_{aa'}D_{ab'}D_{ac'}\cdots D_{am'}}}{\sqrt[n]{r_x' D_{ab}D_{ac}\cdots D_{an}}} \tag{1.45}$$

The inductance of the strand *a* is then given by

$$L_a = \frac{\lambda_a}{(I/n)} = 2n\times10^{-7}\ln\frac{\sqrt[m]{D_{aa'}D_{ab'}D_{ac'}\cdots D_{am'}}}{\sqrt[n]{r_x' D_{ab}D_{ac}\cdots D_{an}}} \tag{1.46}$$

In a similar way the inductances of the other conductors are also obtained. For example,

$$\begin{aligned}L_b &= 2n\times10^{-7}\ln\frac{\sqrt[m]{D_{ba'}D_{bb'}D_{bc'}\cdots D_{bm'}}}{\sqrt[n]{r_x' D_{ab}D_{bc}\cdots D_{bn}}}\\ L_c &= 2n\times10^{-7}\ln\frac{\sqrt[m]{D_{ca'}D_{cb'}D_{cc'}\cdots D_{cm'}}}{\sqrt[n]{r_x' D_{ac}D_{bc}\cdots D_{cn}}}\end{aligned} \tag{1.47}$$

The average inductance of any one of the strands in the group of conductor *x* is then

$$L_{av,x} = \frac{L_a + L_b + L_c + \cdots + L_n}{n} \tag{1.48}$$

Conductor *x* is composed of *n* strands that are electrically parallel. Even though the inductance of the different strand is different, the average inductance of all of them is the same as $L_{av,\,x}$. Assuming that the average inductance given above is the inductance of *n* parallel strands, the total inductance of the conductor *x* is

$$L_x = \frac{L_{av,x}}{n} = \frac{L_a + L_b + L_c + \cdots + L_n}{n^2} \tag{1.49}$$

Substituting the values of L_a , $L_b$etc. in the above equation we get

$$L_x = 2\times10^{-7}\ln\frac{GMD}{GMR_x} \tag{1.50}$$

Where the **geometric mean distance** (*GMD*) and the **geometric mean radius** (*GMR*) are given respectively by

$$GMD = \sqrt[mn]{(D_{aa'}D_{ab'}D_{ac'}\dots D_{am'})\cdots\cdots(D_{na'}D_{nb'}D_{nc'}\cdots D_{nm'})} \tag{1.51}$$

$$GMR_x = \sqrt[n^2]{(r_x' D_{ab}D_{ac}\dots D_{an})\cdots\cdots(r_x' D_{na}D_{nb}\dots D_{nn-1})} \tag{1.52}$$

The inductance of the conductor *y* can also be similarly obtained. The geometric mean radius GMR_y will be different for this conductor. However the geometric mean distance will remain the same.

Double Circuit Three Phase Lines

It is common practice to build double-circuit three-phase lines so as to increase transmission reliability at somewhat enhanced cost. From the point of view of power transfer from one end of the line to the other, it is desirable to build the two lines with as low an inductance/phase as possible. In order to achieve this, self GMD should be made high and mutual GMD should be made low. Therefore, the individual conductors of a phase should be kept as far apart as possible (for high self GMD), while the distance between phases be kept as low as permissible (for low mutual GMD).

Figure 1.10 shows the three sections of the transposition cycle of two parallel circuit three-phase lines with vertical spacing (it is a very commonly used configuration)

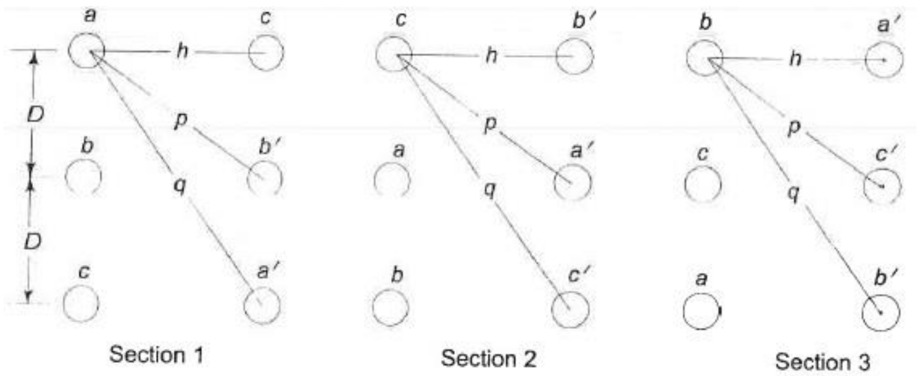


Fig. 1.10 Arrangement of conductors of a double-circuit three-phase line

It may be noted here that conductors a and a' in parallel compose phase a and similarly b and b' compose phase b and c and c' compose phase c. In order to achieve high self GMD the conductors of two phases are placed diametrically opposite to each other and those of the third phase are horizontally opposite to each other. Applying the method of GMD, the equivalent equilateral spacing is

$$D_{eq} = (D_{ab}D_{bc}D_{ca})^{1/3} \tag{1.53}$$

D_{ab} = mutual GMD between phases a and b in section 1 of the transposition cycle

$= (D\times P\times D\times P)^{1/4} = (D\times P)^{1/2}$

D_{bc} = mutual GMD between phases b and c in section 1 of the transposition cycle

$= (D\times P)^{1/2}$

D_{ca} = mutual GMD between phases c and a in section 1 of the transposition cycle

= $(2D\times h)^{1/2}$

Hence

$$D_{eq} = 2^{1/6}D^{1/2}P^{1/3}h^{1/6} \tag{1.54}$$

It may be noted here that D_{eq} remains the same in each section of the transposition cycle, as the conductors of each parallel circuit rotate cyclically, so do D_{ab}, D_{bc} and D_{ca}.

Self GMD in section 1 of phase a (i.e., conductors a and a') is

$$D_{sa} = (r'qr'q)^{1/4} = (r'q)^{1/2}$$

Self GMD in section 1 of phase b and c respectively are

$$D_{sb} = (r'hr'h)^{1/4} = (r'h)^{1/2}$$

$$D_{sc} = (r'qr'q)^{1/4} = (r'q)^{1/2}$$

Equivalent self GMD

$$D_s = (D_{sa}D_{sb}D_{sc})^{1/3}$$

$$D_s = (r')^{1/2}q^{1/3}h^{1/6} \tag{1.55}$$

Because of the cyclic rotation of conductors of each parallel circuit over the transposition cycle, D_s also remains the same in each transposition section.

The inductance Per Phase is

$$L = 2\times 10^{-7} ln\frac{D_{eq}}{D_s}$$

$$L = 2\times 10^{-7} ln\frac{2^{1/6}D^{1/2}P^{1/3}h^{1/6}}{(r')^{1/2}q^{1/3}h^{1/6}}$$

$$L = 2\times 10^{-7}\ln\left[\ 2^{1/6}\left(\frac{D^{1/2}}{(r')^{1/2}}\right)\left(\frac{P^{1/3}}{q^{1/3}}\right)\right] \qquad {}^{H}\!/_{ph}\!/_{m} \tag{1.56}$$

Bundled Conductors

So far we have discussed three-phase systems that have only one conductor per phase. However for extra high voltage lines corona causes a large problem if the conductor has only one conductor per phase. Corona occurs when the surface potential gradient of a conductor exceeds the dielectric strength of the surrounding air. This causes ionization of the area near the conductor. Corona produces power loss. It also causes interference with communication channels. Corona manifests itself with a hissing sound and ozone discharge. Since most long distance power lines in India are either 220 kV or 400 kV, avoidance of the occurrence of corona is desirable.

The high voltage surface gradient is reduced considerably by having two or more conductors per phase in close proximity. This is called conductor bundling. The conductors are bundled in groups of two, three or four as shown in Fig. 1.11. The conductors of a bundle are separated at regular intervals with spacer dampers that prevent clashing of the conductors and prevent them from swaying in the wind. They also connect the conductors in parallel.

The geometric mean radius (GMR) of two-conductor bundle is given by

$$D_{s,2b} = \sqrt[4]{(D_s \times d)^2} = \sqrt{D_s \times d} \tag{1.57}$$

Where D_s is the *GMR* of conductor. The GMR for three-conductor and four-conductor bundles are given respectively by

$$D_{s,3b} = \sqrt[9]{(D_s \times d \times d)^3} = \sqrt[3]{D_s \times d^2} \tag{1.58}$$

$$D_{s,4b} = \sqrt[16]{\left(D_s \times d \times d \times \sqrt{2}d\right)^4} = 1.09\sqrt[4]{D_s \times d^3} \tag{1.59}$$

The inductance of the bundled conductor is then given by

$$L = 2\times10^{-7} \ln \frac{GMD}{D_{s,nb}} \quad \text{where n=2,3 ....} \tag{1.60}$$

where the geometric mean distance is calculated assuming that the center of a round conductor is the same as that of the center of the bundle.

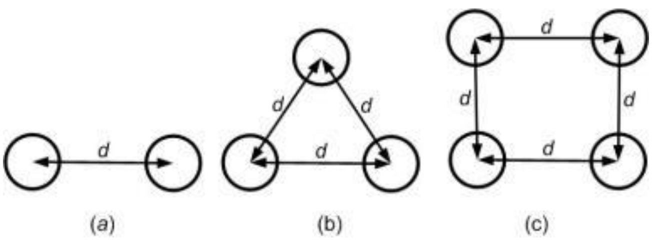


Fig. 1.11 Bundled conductors: (a) 2-conductor, (b) 3-conductor and (c) 4-conductor bundles

PS-II, UNIT - II

Capacitance of Overhead Transmission Lines

Learning objectives:

- Derive the expression for capacitance for single and three phase systems
- Describe the effect of earth on capacitance of transmission lines

Syllabus:

Calculation of capacitance for 2 wire and 3 wire systems, effect of ground on capacitance - method of images, capacitance calculations for symmetrical and asymmetrical single and three phase, introduction of double circuit lines, Numerical Problems.

Learning outcomes:

Students will be able to

- obtain the expression for capacitance for single and three phase systems
- demonstrate the effect of earth on capacitance.

Capacitance of Overhead Transmission Lines

Capacitance in a transmission line results due to the potential difference between the conductors. The conductors get charged in the same way as the parallel plates of a capacitor. Capacitance between two parallel conductors depends on the size and the spacing between the conductors. Usually the capacitance is neglected for the transmission lines that are less than 50 miles (80 km) long. However the capacitance becomes significant for longer lines with higher voltage. In this section we shall derive the line capacitance of different line configuration.

Capacitance of a Straight Conductor

Consider the round conductor shown in Fig. 1.12. The conductor has a radius of r and carries a charge of q coulombs. The capacitance C is the ratio of charge q of the conductor to the impressed voltage, i.e.,

$$C = \frac{q}{V} \tag{1.61}$$

The charge on the conductor gives rise to an electric field with radial flux lines where the total electric flux is equal to the charge on the conductor. By Gauss's law, the electric flux density at a cylinder of radius x when the conductor has a length of 1 m is

$$D = \frac{q}{A} = \frac{q}{2\pi x} \text{ C/m}^2 \tag{1.62}$$

The electric filed intensity is defined as the ratio of electric flux density to the permittivity of the medium. Therefore

$$E = \frac{q}{2\pi x \varepsilon_0} \text{ V/m} \tag{1.63}$$

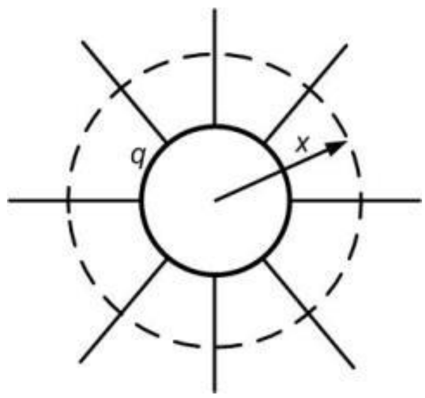


Fig. 1.12 Cylindrical conductor with radial flux lines.

Now consider the long straight conductor of Fig. 1.13 that is carrying a positive charge q C/m. Let two pointsP_1 and P_2 be located at distances D_1 and D_2 respectively from the center of the conductor. The conductor is an equipotential surface in which we can assume that the uniformly distributed charge is concentrated at the canter of the conductor. The potential difference V_{12} between the points P_1 and P_2 is the work done in moving a unit of charge from P_2 to P_1 . Therefore the voltage drop between the two points can be computed by integrating the field intensity over a radial path between the equipotential surfaces, i.e.,

$$V_{12} = \int_{D_1}^{D_2} E dx = \int_{D_1}^{D_2} \frac{q}{2\pi x \varepsilon_0} dx = \frac{q}{2\pi \varepsilon_0} \ln \frac{D_2}{D_1} \text{ V} \tag{1.64}$$

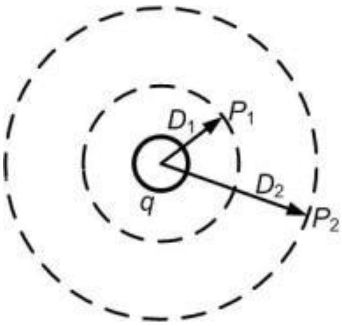


Fig. 1.13 Path of integration between two points external to a round straight conductor.

Capacitance of a Single-Phase Line

Consider the single-phase line consisting of two round conductors as shown in Fig. 1.14. The separation between the conductors is *D*. Let us assume that conductor 1 carries a charge of q_1 C/m while conductor 2 carries a charge q_2 C/m. The presence of the second conductor and the ground will disturb field of the first conductor. However we assume that the distance of separation between the conductors is much larger compared to the radius of the conductor and the height of the conductor is much larger than *D* for the ground to disturb the flux. Therefore the distortion is small and the charge is uniformly distributed on the surface of the conductor.

Assuming that the conductor 1 alone has the charge q_1 , the voltage between the conductors is

$$V_{12}(q_1) = \frac{q_1}{2\pi \varepsilon_0} \ln \frac{D_2}{r_1} \text{ V} \tag{1.65}$$

Similarly if the conductor 2 alone has the charge q_2 , the voltage between the conductors is

$$V_{21}(q_2) = \frac{q_2}{2\pi \varepsilon_0} \ln \frac{D}{r_2} \text{ V}$$

The above equation implies that

$$V_{12}(q_2) = \frac{q_2}{2\pi \varepsilon_0} \ln \frac{r_2}{D} \text{ V} \tag{1.66}$$

From the principle of superposition we can write

$$V_{12} = V_{12}(q_1) + V_{12}(q_2) = \frac{q_1}{2\pi \varepsilon_0} \ln \frac{D}{r_1} + \frac{q_2}{2\pi \varepsilon_0} \ln \frac{r_2}{D} \text{ V} \tag{1.67}$$

For a single-phase line let us assume that q_1 (= $-q_2$) is equal to q . We therefore have

$$V_{12} = \frac{q}{2\pi \varepsilon_0} \ln \frac{D}{r_1} - \frac{q}{2\pi \varepsilon_0} \ln \frac{r_2}{D} = \frac{q}{2\pi \varepsilon_0} \ln \frac{D^2}{r_1 r_2} \text{ V} \tag{1.68}$$

Assuming $r_1 = r_2 = r_3$, we can rewrite (1.68) as

$$V_{12} = \frac{q}{\pi \varepsilon_0} \ln \frac{D}{r} \text{ V} \tag{1.69}$$

Therefore from (1.61) the capacitance between the conductors is given by

$$C_{12} = \frac{\pi \varepsilon_0}{\ln(D/r)} \text{ V F/m} \tag{1.70}$$

The above equation gives the capacitance between two conductors. For the purpose of transmission line modelling, the capacitance is defined between the conductor and neutral. This is shown in Fig. 1.14. Therefore the value of the capacitance is given from Fig. 1.13 as

$$C = 2C_{12} = \frac{2\pi \varepsilon_0}{\ln(D/r)} \text{ F/m} \tag{1.71}$$

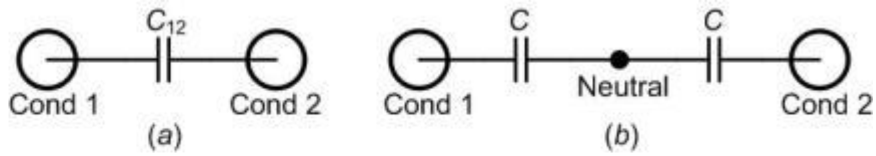


Fig. 1.14 (a) Capacitance between two conductors and (b) equivalent capacitance to ground.

Capacitance of a Three-Phase Transposed Line

Consider the three-phase transposed line shown in Fig. 1.15. In this the charges on conductors of phases a, b and c are q_a, $q_b$and q_c espectively. Since the system is assumed to be balanced we have

$$q_a + q_b + q_c = 0 \tag{1.72}$$

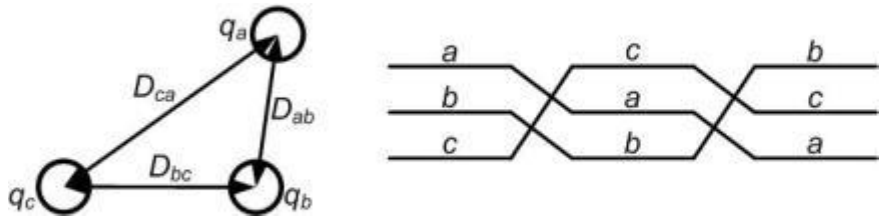


Fig. 1.15 Charge on a three-phase transposed line.

Using superposition, the voltage V_{ab} for the first, second and third sections of the transposition are given respectively as

$$V_{ab}(1) = \frac{1}{2\pi \varepsilon_0}\left(q_a \ln\frac{D_{ab}}{r} + q_b \ln\frac{r}{D_{ab}} + q_c \ln\frac{D_{bc}}{D_{ca}}\right) \text{ V} \tag{1.73}$$

$$V_{ab}(2) = \frac{1}{2\pi \varepsilon_0}\left(q_a \ln\frac{D_{bc}}{r} + q_b \ln\frac{r}{D_{bc}} + q_c \ln\frac{D_{ca}}{D_{ca}}\right) \text{ V} \tag{1.74}$$

$$V_{ab}(3) = \frac{1}{2\pi \varepsilon_0}\left(q_a \ln\frac{D_{ca}}{r} + q_b \ln\frac{r}{D_{ca}} + q_c \ln\frac{D_{ab}}{D_{bc}}\right) \text{ V} \tag{1.75}$$

Then the average value of the voltage is

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{D_{ab}D_{bc}D_{ca}}{r^3} + q_b \ln\frac{r^3}{D_{ab}D_{bc}D_{ca}} + q_c \ln\frac{D_{ab}D_{bc}D_{ca}}{D_{ab}D_{bc}D_{ca}}\right) \text{ V} \tag{1.76}$$

This implies

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{\sqrt[3]{D_{ab}D_{bc}D_{ca}}}{r} + q_b \ln\frac{r}{\sqrt[3]{D_{ab}D_{bc}D_{ca}}}\right) \text{ V} \tag{1.77}$$

The *GMD* of the conductors is given in (1.42). We can therefore write

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{GMD}{r} + q_b \ln\frac{r}{GMD}\right) \text{ V} \tag{1.78}$$

Similarly the voltage V_{ac} is given as

$$V_{ac} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{GMD}{r} + q_c \ln\frac{r}{GMD}\right) \tag{1.79}$$

Adding (1.78) and (1.79) and using (1.72) we get

$$\begin{aligned} V_{ab} + V_{ac} &= \frac{1}{2\pi\varepsilon_0}\left[2q_a \ln\frac{GMD}{r} + (q_b + q_c)\ln\frac{r}{GMD}\right] \\ &= \frac{1}{2\pi\varepsilon_0}\left[2q_a \ln\frac{GMD}{r} - q_a \ln\frac{r}{GMD}\right] = \frac{3}{2\pi\varepsilon_0} q_a \ln\frac{GMD}{r} \end{aligned} \tag{1.80}$$

For a set of balanced three-phase voltages

$$\begin{aligned} V_{ab} &= V_{an}\angle 0° - V_{an}\angle -120° \\ V_{ac} &= V_{an}\angle 0° - V_{an}\angle -240° \end{aligned}$$

Therefore we can write

$$V_{ab} + V_{ac} = 2V_{an}\angle 0° - V_{an}\angle -120° - V_{an}\angle -240° = 2V_{an}\angle 0° \text{ V} \tag{1.81}$$

Combining (1.80) and (1.81) we get

$$V_{an} = \frac{1}{2\pi\varepsilon_0} q_a \ln\frac{GMD}{r} \text{ V} \tag{1.82}$$

Therefore the capacitance to neutral is given by

$$C = \frac{q_a}{V_{ax}} = \frac{2\pi\,\varepsilon_0}{\ln\left(GMD/r\right)} \quad \text{F/m} \tag{1.83}$$

For bundles conductor

$$C = \frac{2\pi\,\varepsilon_0}{\ln\left(GMD/r\right)}$$

where

$$\begin{aligned} D_b &= \sqrt{\pi d} \ \text{ for 2 bundle} \\ &= \sqrt[3]{\pi d^2} \ \text{ for 3 bundle} \\ &= {}^{1.094}\sqrt{\pi d^3} \ \text{ for 4 bundle conductors} \end{aligned}$$

Effect of Earth on the Calculation of Capacitance

Earth affects the calculation of capacitance of three-phase lines as its presence alters the electric field lines. Usually the height of the conductors placed on transmission towers is much larger than the spacing between the conductors. Therefore the effect of earth can be neglected for capacitance calculations, especially when balanced steady state operation of the power system is considered. However for unbalanced operation when the sum of the three line currents is not zero, the effect of earth needs to be considered.

In calculating the capacitance of transmission lines, the presence of earth was ignored, so far. The effect of earth on capacitance can be conveniently taken into account by the method of images.

Method of images

The electric field of transmission line conductors must conform to the presence of the earth below. The earth for this purpose may be assumed to be a perfect conducting horizontal sheet of infinite extent which therefore acts like an equipotential surface.

The electric field of two long conductors charged +q and –q per unit is such that it has a zero potential plane midway between the conductors as shown in Fig. If a conducting sheet of infinite dimensions is placed at the zero potential plane, the electric field remains undisturbed. Further, if the conductor carrying charge -q is now removed, the electric field above the conducting sheet stays intact, while that below it vanishes. Using these well known results in reverse, we may equivalently replace the presence of ground below a charged conductor by a fictitious conductor having equal and opposite charge and located as far below the surface of ground as the overhead conductor above it-such a fictitious conductor is the mirror image of the overhead conductor. This method of creating the same electric field as in the presence of earth is known as the method of images originally suggested by Lord Kelvin.

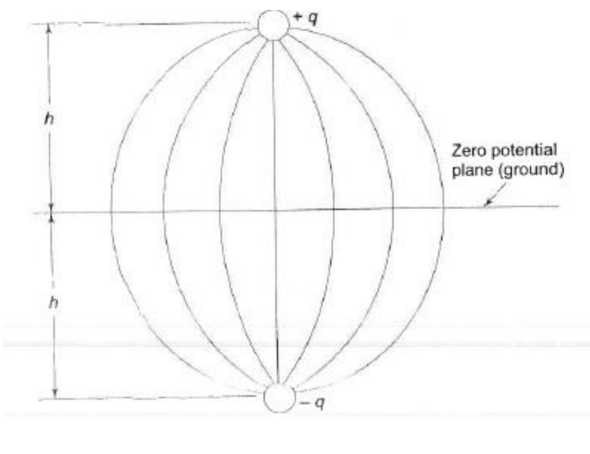


Fig. 1.16 Electric field of two long, parallel, oppositely charged conductors

Capacitance of a Single Phase Line

Consider a single-phase line as shown in Fig. lt is required to calculate it's capacitance taking the presence of earth into account by the method of images described above. The equation for the voltage drop V_{ab} as determined by the two charged conductors 'a' and 'b', and their images a' and b' can be written as follows:

$$V_{ab} = \frac{1}{2\pi k}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D} + q_{c'} \ln\frac{(4h^2+D^2)^2}{2h} + q_{b'} \ln\frac{2h}{(4h^2+D^2)^2}\right] \qquad (1.84)$$

Substituting the values of different charges and simplifying, we get

$$V_{ab} = \frac{q}{\pi k}\left[\ln\frac{2hD}{r(4h^2+D^2)^{1/2}}\right] \qquad (1.85)$$

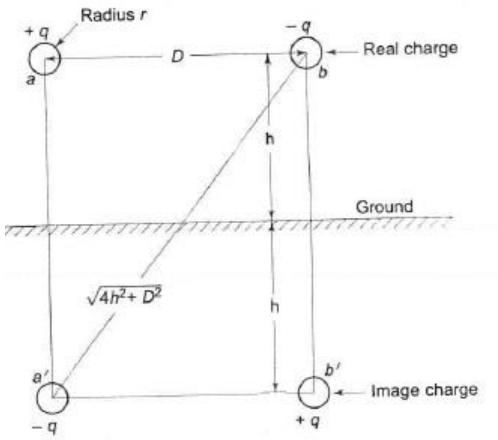


Fig. 1.17 Single phase transmission line with images

It immediately follows that

$$C_{ab} = \frac{\pi k}{\ln\frac{D}{r(1+(D^2/4h^2))^{1/2}}} \qquad (1.86)$$

and

$$C_{n} = \frac{2\pi k}{\ln\frac{D}{r(1+(D^2/4h^2))^{1/2}}} \qquad (1.87)$$

Capacitance of a Three-Phase Line considering earth effect

The method of images can similarly be applied for the calculation of capacitance of a three-phase line, shown in figure. The line is considered to be fully transposed. The conductors a, b, and c carry the charges q_a, q_b, q_c and occupy positions 1, 2, and 3, respectively, in the first cycle of transposition cycle. The effect of earth is simulated by image conductors with charges $-q_a$, $-q_b$, and $-q_c$ respectively, as shown.

The equations for the three sections of the transposition cycle can be written for the voltage drop V_{ab} as determined by the charged conductors and their images. With conductor a in position 1, b in position2, and c in position 3,

$$V_{ab} = \frac{1}{2\pi k}\left[q_a\left(\ln\frac{D_{12}}{r} - \ln\frac{h_{12}}{h_1}\right) + q_b\left(\ln\frac{r}{D_{12}} - \ln\frac{h_2}{h_{12}}\right) + q_c\left(\ln\frac{D_{23}}{D_{31}} - \ln\frac{h_{23}}{h_{31}}\right)\right] \qquad (1.88)$$

Similar equations for V_{ab} can be written for the second and third sections of the transposition cycle. If the fairly accurate assumption of constant charge per unit length of the conductor throughout the transmission cycle is made, the average value of V_{ab} for the three sections of the cycle is given by

$$V_{ab} = \frac{1}{2\pi k}\left[q_a\left(\ln\frac{D_{eq}}{r} - \ln\frac{(h_{12}h_{23}h_{31})^{1/3}}{(h_1h_2h_3)^{1/3}}\right) + q_b\left(\ln\frac{r}{D_{eq}} - \ln\frac{(h_1h_2h_3)^{1/3}}{(h_{12}h_{23}h_{31})^{1/3}}\right)\right] \qquad (1.89)$$

Where $D_{eq} = (D_{12}\,D_{23}\,D_{31})^{1/3}$

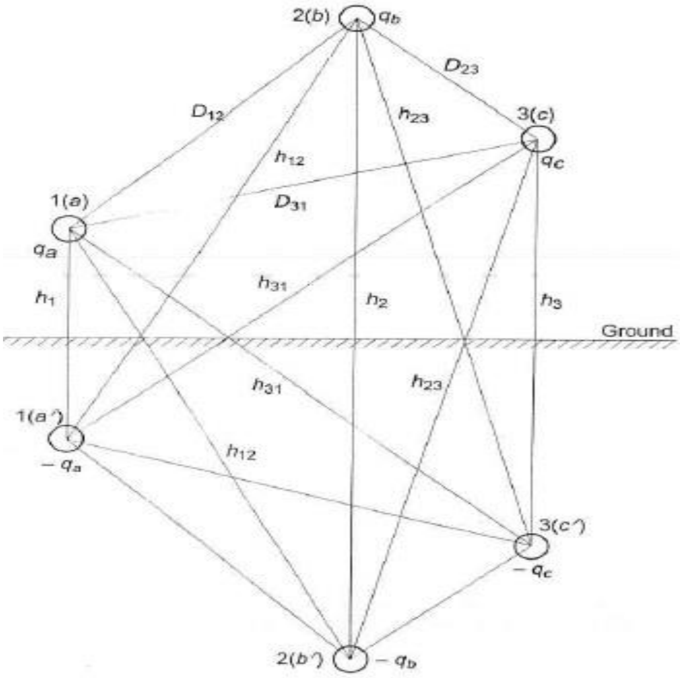


Fig. 1.18 Three-phase line with images

The equation for the average value of the phasor V_{ac} is found in a similar manner. For balanced three phase voltages $V_{ab} + V_{ac} = 3V_{an}$ and $q_a + q_b + q_c = 0$, we ultimately obtained the following expression for the capacitance to neutral.

$$C_n = \frac{2\pi k}{\ln\frac{D_{eq}}{r} - \ln\left(\frac{(h_{12}h_{23}h_{31})^{1/3}}{(h_1h_2h_3)^{1/3}}\right)} \qquad (1.90)$$

Comparing capacitance with and without earth effect, it is evident that the effect of earth is to increase the capacitance of a line. If the conductors are high above earth compared to the distances among them, the effect of earth on the capacitance of three-phase lines can be neglected

PS-II: UNIT – III

Performance of Short and Medium Length Transmission Lines

Learning Objectives:

- Able to understand the insight into specific transmission lines short and medium type which would have application in medium and high voltage power transmission systems.

Learning outcomes:

At the end of this chapter students will be able to

- describe how transmission lines are classified viz. Short, Medium and Long transmission lines.
- obtain the ABCD parameters of short and medium transmission lines.
- Analyze the Performance of short and medium transmission lines.

Syllabus:

Classification of Transmission Lines - Short, medium, long line and their model representations - Nominal-T, Nominal-Pie and A, B, C, D Constants for symmetrical & Asymmetrical Networks, Numerical Problems. Mathematical Solutions to estimate regulation and efficiency of all types of lines - Numerical Problems.

UNIT – III

Performance of Short and Medium Length Transmission Lines

Introduction

The important considerations in the design and operation of a transmission line are the determination of voltage drop, line losses and efficiency of transmission. These values are greatly influenced by the line constants *R*, *L* and *C* of the transmission line. For instance, the voltage drop in the line depends upon the values of above three line constants. Similarly, the resistance of transmission line conductors is the most important cause of power loss in the line and determines the transmission efficiency. In this chapter, we shall develop formulas by which we can calculate voltage regulation, line losses and efficiency of transmission lines. These formulas are important for two principal reasons. Firstly, they provide an opportunity to understand the effects of the parameters of the line on bus voltages and the flow of power. Secondly, they help in developing an overall understanding of what is occurring on electric power system.

2.1 Classification of Overhead Transmission Lines

A transmission line has *three constants *R*, *L* and *C* distributed uniformly along the whole length of the line. The resistance and inductance form the series impedance. The capacitance existing between conductors for 1-phase line or from a conductor to neutral for a 3-phase line forms a shunt path throughout the length of the line. Therefore, capacitance effects introduce complications in transmission line calculations. Depending upon the manner in which capacitance is taken into account, the overhead transmission lines are classified as:

i. **Short transmission lines:**

When the length of an overhead transmission line is up to about 50 km and the line voltage is comparatively low (< 20 kV), it is usually considered as a short transmission line. Due to smaller length and lower voltage, the capacitance effects are small and hence can be neglected. Therefore, while studying the performance of a short transmission line, only resistance and inductance of the line are taken into account.

ii. **Medium transmission lines:**

When the length of an overhead transmission line is about 50- 150 km and the line voltage is moderately high (>20 kV < 100 kV), it is considered as a medium transmission line. Due to sufficient length and voltage of the line, the capacitance effects are taken into account. For purposes of calculations, the distributed capacitance of the line is divided and lumped in the form of condensers shunted across the line at one or more points.

iii. **Long transmission lines:**

When the length of an overhead transmission line is more than 150 km and line voltage is very high (> 100 kV), it is considered as a long transmission line. For the treatment of such a line, the line constants are considered uniformly distributed over the whole length of the line and rigorous methods are employed for solution.

It may be emphasized here that exact solution of any transmission line must consider the fact that the constants of the line are not lumped but are distributed uniformly throughout the length of the line. However, reasonable accuracy can be obtained by considering these constants as lumped for short and medium transmission lines.

2.2 Important Terms

While studying the performance of a transmission line, it is desirable to determine its voltage regulation and transmission efficiency. We shall explain these two terms in turn.

i. **Voltage regulation:**

When a transmission line is carrying current, there is a voltage drop in the line due to resistance and inductance of the line. The result is that receiving end voltage (V_R) of the line is generally less than the sending end voltage (V_S). This voltage drop ($V_S - V_R$) in the line is expressed as a percentage of receiving end voltage V_R and is called voltage regulation.

The difference in voltage at the receiving end of a transmission line between conditions of no load and full load is called **voltage regulation** and is expressed as a percentage of the receiving end voltage.

Mathematically,

$$\%\text{ age Voltage regulation} = \frac{V_S - V_R}{V_R} * 100$$

Obviously, it is desirable that the voltage regulation of a transmission line should be low *i.e.*, the increase in load current should make very little difference in the receiving end voltage.

ii. Transmission efficiency:

The power obtained at the receiving end of a transmission line is generally less than the sending end power due to losses in the line resistance.

The ratio of receiving end power to the sending end power of a transmission line is known as the **transmission efficiency** *of the line i.e.*

$$\%\text{ age Transmission efficiency.}\,\eta_r = \frac{\text{Receiving end power}}{\text{Sending end power}} * 100$$

$$= \frac{V_R I_R \cos\phi_R}{V_S I_S \cos\phi_S} * 100$$

Where V_R, I_R and $\cos\phi_R$ are the receiving end voltage, current and power factor while V_S, I_S and $\cos\phi_S$ are the corresponding values at the sending end.

2.3 Performance of Single Phase Short Transmission Lines

As stated earlier, the effects of line capacitance are neglected for a short transmission line. Therefore, while studying the performance of such a line, only resistance and inductance of the line are taken into account. The equivalent circuit of a single phase short transmission line is shown in Fig.1 (*i*). Here, the total line resistance and inductance are shown as concentrated or lumped instead of being distributed. The circuit is a simple a.c. series circuit.

Let

$$\text{I} = \text{load current}$$

$$R = \text{loop resistance } i.e., \text{resistance of both conductors}$$

$$X_L = \text{ loop reactance}$$

$$V_r = \text{receiving end voltage}$$

$$\cos\emptyset_R = \text{receiving end power factor (lagging)}$$

$$V_S = \text{Sending end voltage}$$

$$\cos\emptyset_S = \text{Sending end power factor}$$

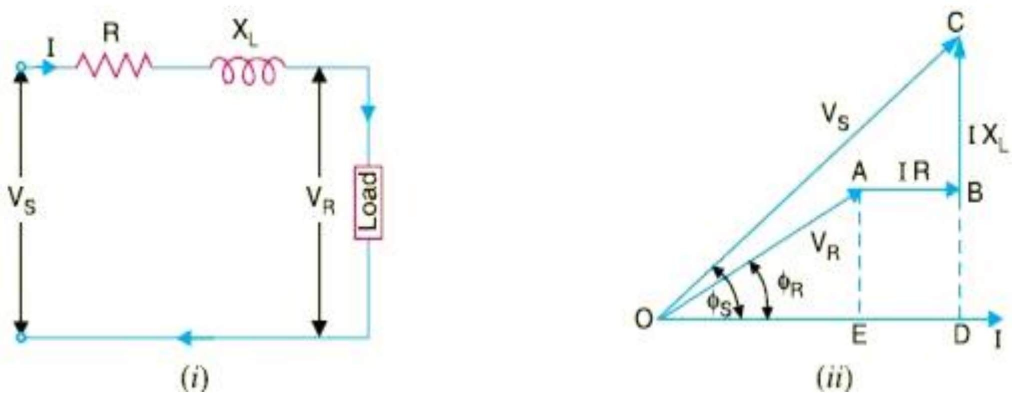


Fig. 1

Phasor diagram: Current I is taken as the reference phasor. OA represents the receiving end voltage V_R leading I by $\cos\phi_R$. AB represents the drop IR in phase with I. BC represents the inductive drop IXL and leads I by 90^0. OC represents the sending end voltage V_S and leads I by $\cos\phi_S$.

The phasor diagram of the line for lagging load power factor is shown in Fig. 1 (*ii*). From the right angled triangle *ODC*, we get,

$$(OC)^2 = (OD)^2 + (DC)^2$$

or

$$V_S^2 = (OE + ED)^2 + (DB + BC)^2$$

$$= (V_R \cos\phi_R + IR)^2 + (V_R \sin\phi_R + IX_L)^2$$

$$\therefore \quad V_S = \sqrt{(V_R \cos\phi_R + IR)^2 + (V_R \sin\phi_R + IX_L)^2}$$

$$\text{\%age Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\text{Sending end } p.f., \cos\phi_S = \frac{OD}{OC} = \frac{V_R \cos\phi_R + IR}{V_S}$$

$$\text{Power delivered} = V_R I_R \cos\phi_R$$

$$\text{Line losses} = I^2 R$$

$$\text{Power sent out} = V_R I_R \cos\phi_R + I^2 R$$

$$\text{\%age Transmission efficiency} = \frac{\text{Power delivered}}{\text{Power sent out}} \times 100$$

$$= \frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + I^2 R} \times 100$$

An approximate expression for the sending end voltage VS can be obtained as follows. Draw perpendicular from B and C on OA produced as shown in Fig. 2. Then OC is nearly equal to OF i.e.,

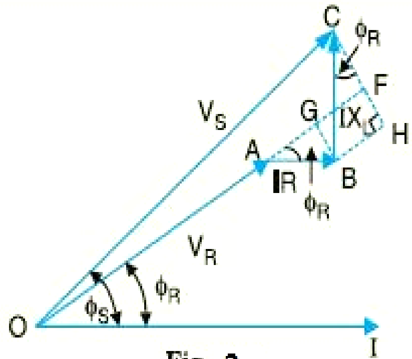


Fig. 2

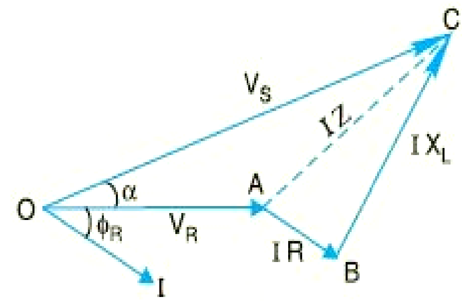


Fig. 3

$$
\begin{aligned}
OC &= OF = OA + AF = OA + AG + GF \\
&= OA + AG + BH \\
\therefore \qquad V_S &= V_R + IR\cos\phi_R + IX_L\sin\phi_R
\end{aligned}
$$

Solution in complex notation:

It is often convenient and profitable to make the line calculations in complex notation. Taking $\overrightarrow{V_R}$ as the reference phasor, draw the phasor diagram as shown in Fig 3. It is clear that $\overrightarrow{V_S}$ is the phasor sum of $\overrightarrow{V_R}$ and $\vec{I}\vec{Z}$.

$$
\begin{aligned}
\overrightarrow{V_R} &= V_R + j\,0 \\
\vec{I} &= \vec{I}\ \angle -\phi_R = I(\cos\phi_R - j\sin\phi_R) \\
\vec{Z} &= R + jX_L \\
\overrightarrow{V_S} &= \overrightarrow{V_R} + \vec{I}\,\vec{Z} \\
&= (V_R + j\,0) + I(\cos\phi_R - j\sin\phi_R)(R + j\,X_L)
\end{aligned}
$$

$$
= (V_R + IR\cos\phi_R + IX_L\sin\phi_R) + j\,(IX_L\cos\phi_R - IR\sin\phi_R)
$$

$$
\therefore \qquad V_S = \sqrt{(V_R + IR\cos\phi_R + IX_L\sin\phi_R)^2 + (IX_L\cos\phi_R - IR\sin\phi_R)^2}
$$

The second term under the root is quite small and can be neglected with reasonable accuracy. Therefore, approximate expression for V_S becomes:

$$V_S = V_R + IR\cos\emptyset_R + IX_L\sin\emptyset_R$$

The following points may be noted :

i. The approximate formula for $V_S = V_R + IR\cos\emptyset_R + IX_L\sin\emptyset_R$ gives fairly correct results for lagging power factors. However, appreciable error is caused for leading power factors. Therefore, approximate expression for *VS* should be used for lagging p.f. only.

ii. The solution in complex notation is in more presentable form.

2.4 Three-Phase Short Transmission Lines

For reasons associated with economy, transmission of electric power is done by 3-phase system. This system may be regarded as consisting of three single phase units, each wire transmitting one-third of the total power. As a matter of convenience, we generally analyse 3-phase system by considering one phase only. Therefore, expression for regulation, efficiency etc. derived for a single phase line can also be applied to a 3-phase system. Since only one phase is considered, phase values of 3-phase system should be taken. Thus, V_S and V_R are the phase voltages, whereas *R* and X_L are the resistance and inductive reactance per phase respectively.

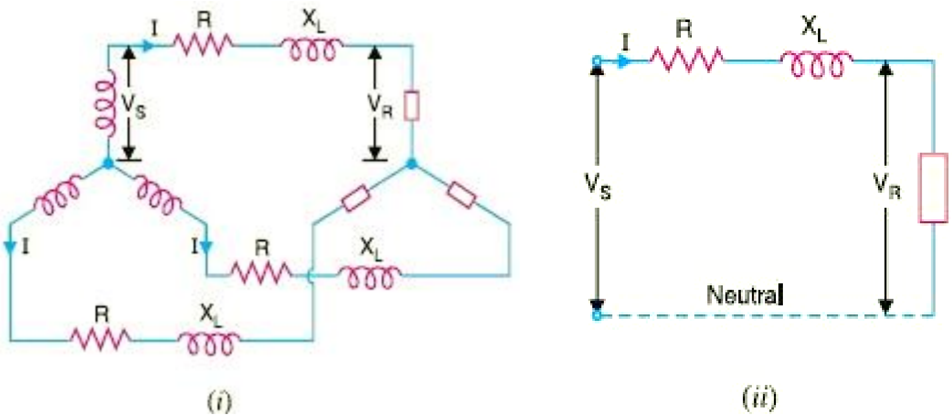


Fig. 4

Fig. 4 (*i*) shows a *Y*-connected generator supplying a balanced *Y*-connected load through a transmission line. Each conductor has a resistance of *R* Ω and inductive reactance of $X_L \Omega$. Fig. 4 (*ii*) shows one phase separately. The calculations can now be made in the same way as for a single phase line.

2.5 Effect of Load power factor on Regulation and Efficiency

The regulation and efficiency of a transmission line depend to a considerable extent upon the power factor of the load.

<u>Effect on regulation</u>: The expression for voltage regulation of a short transmission line is given by:

$$\% \, Voltage \ Regulation \ = \frac{IR\cos\emptyset_R + IX_L\sin\emptyset_R}{V_R} * 100 \qquad (\textit{for lagging p.f.})$$

$$\% \, Voltage \ Regulation \ = \frac{IR\cos\emptyset_R - IX_L\sin\emptyset_R}{V_R} * 100 \qquad (\textit{for leading p.f.})$$

The following conclusions can be drawn from the above expressions:

i. When the load p.f. is lagging or unity or such leading that $IR\cos\emptyset_R > IX_L\sin\emptyset_R$ then voltage regulation is positive *i.e.*, receiving end voltage V_R will be less than the sending end voltage V_S

ii. For a given V_R and I, the voltage regulation of the line increases with the decrease in p.f. for lagging loads.

iii. When the load p.f. is leading to this extent that $IX_L\sin\emptyset_R > IR\cos\emptyset_R$ then voltage regulation is negative *i.e.* the receiving end voltage V_R is more than the sending end voltage V_S.

iv. For a given V_R and I, the voltage regulation of the line decreases with the decrease in p.f. for leading loads.

<u>Effect on transmission efficiency</u>

The power delivered to the load depends upon the power factor.

$$P = V_R * I_R\cos\emptyset_R \qquad \text{(For 1-phase line)}$$

$$I = \frac{P}{V_R\cos\emptyset_R}$$

$$P = 3 * V_R * I_R\cos\emptyset_R \qquad \text{(For 3-phase line)}$$

$$I = \frac{P}{3{*}V_R\cos\emptyset_R}$$

It is clear that in each case, for a given amount of power to be transmitted (P) and receiving end voltage (VR), the load current I is inversely proportional to the load power factor $\cos\emptyset_R$. Consequently, with the decrease in load p.f., the load current and hence the line losses are increased. This leads to the conclusion that transmission efficiency of a line decreases with the decrease in load power factor and *vice-versa*,

2.6 Medium Transmission Lines

In short transmission line calculations, the effects of the line capacitance are neglected because such lines have smaller lengths and transmit power at relatively low voltages (< 20 kV).

However, as the length and voltage of the line increase, the capacitance gradually becomes of greater importance. Since medium transmission lines have sufficient length (50-150 km) and usually operate at voltages greater than 20 kV, the effects of capacitance cannot be neglected. Therefore, in order to obtain reasonable accuracy in medium transmission line calculations, the line capacitance must be taken into consideration.

The capacitance is uniformly distributed over the entire length of the line. However, in order to make the calculations simple, the line capacitance is assumed to be lumped or concentrated in the form of capacitors shunted across the line at one or more points. Such a treatment of localizing the line capacitance gives reasonably accurate results. The most commonly used methods (known as localized capacitance methods) for the solution of medium transmissions lines are :

i. End condenser method
ii. Nominal *T* method
iii. Nominal Π method

Although the above methods are used for obtaining the performance calculations of medium lines, they can also be used for short lines if their line capacitance is given in a particular problem.

2.7 End Condenser Method

In this method, the capacitance of the line is lumped or concentrated at the receiving or load end as shown in Fig. 8.

This method of localizing the line capacitance at the load end overestimates the effects of capacitance. In Fig. 8, one phase of the 3-phase transmission line is shown as it is more convenient to work in phase instead of line-to-line values.

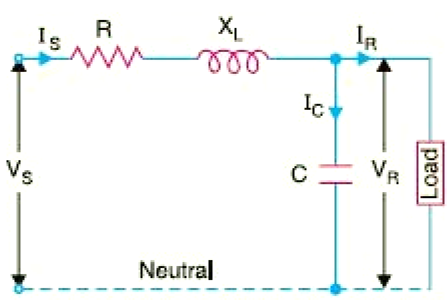


Fig 8

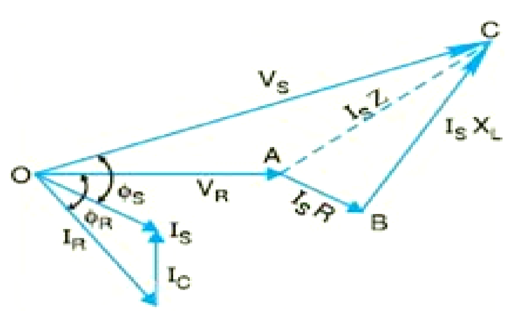


Fig 9

Let

I_R= load current per phase

R = resistance per phase

X_L = inductive reactance per phase

C = capacitance per phase

$\cos\phi_R$ = receiving end power factor (*lagging*)

$V_S =$ sending end voltage per phase

The phasor diagram for the circuit is shown in Fig 9. Taking the receiving end voltage $\overrightarrow{V_R}$ as the reference phasor, we have, $\overrightarrow{V_R} = V_R + j0$

Load current, $\overrightarrow{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

Capacitive current, $\overrightarrow{I_C} = j\,\overrightarrow{V_R}\,\omega C = j\,2\pi f C\,\overrightarrow{V_R}$

The sending end current $\overrightarrow{I_S}$ is the phasor sum of load current $\overrightarrow{I_R}$ and capacitive current $\overrightarrow{I_C}$ *i.e.*,

$$\begin{aligned}\overrightarrow{I_S} &= \overrightarrow{I_R} + \overrightarrow{I_C}\\ &= I_R(\cos\phi_R - j\sin\phi_R) + j\,2\pi f C\,V_R\\ &= I_R\cos\phi_R + j(-I_R\sin\phi_R + 2\pi f C V_R)\end{aligned}$$

Voltage drop/phase $= \overrightarrow{I_S}\,\overrightarrow{Z} = \overrightarrow{I_S}\,(R + jX_L)$

Sending end voltage, $\overrightarrow{V_S} = \overrightarrow{V_R} + \overrightarrow{I_S}\,\overrightarrow{Z} = \overrightarrow{V_R} + \overrightarrow{I_S}\,(R + j\,X_L)$

Thus, the magnitude of sending end voltage V_S can be calculated.

$$\%\text{ Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\%\text{ Voltage transmission efficiency} = \frac{\text{Power delivered / phase}}{\text{Power delivered / phase + losses / phase}} \times 100$$

$$= \frac{V_R\,I_R\cos\phi_R}{V_R\,I_R\cos\phi_R + I_S^2\,R} \times 100$$

Limitations:

Although end condenser method for the solution of medium lines is simple to work out calculations, yet it has the following drawbacks:

i. There is a considerable error (about 10%) in calculations because the distributed capacitance has been assumed to be lumped or concentrated.

ii. This method overestimates the effects of line capacitance.

2.8 Nominal - T Method

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Fig. 10. Therefore, in this arrangement, full charging current flows over half the line. In Fig. 11, one phase of 3- phase transmission line is shown as it is advantageous to work in phase instead of line-to-line values.

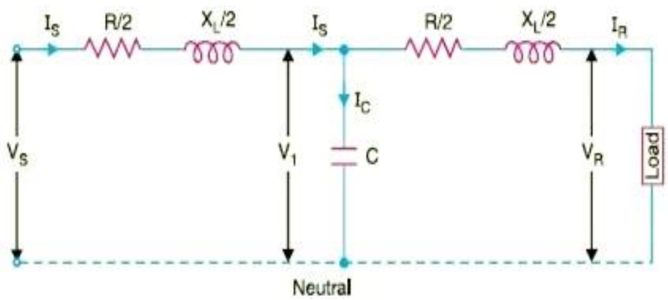


Fig. 10

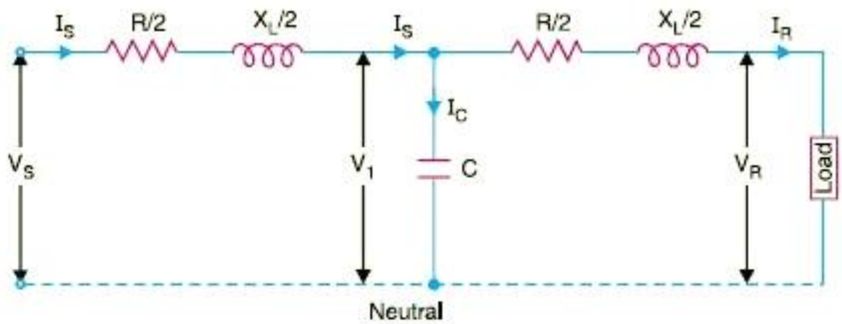


Fig. 11

Let I_R = load current per phase ; R = resistance per phase

X_L = inductive reactance per phase ; C = capacitance per phase

$\cos \phi_R$ = receiving end power factor (*lagging*) ; V_S = sending end voltage/phase

V_1 = voltage across capacitor C

The *phasor diagram for the circuit is shown in Fig. 12. Taking the receiving end voltage $\vec{V_R}$ as the reference phasor, we have,

Receiving end voltage, $\vec{V_R} = V_R + j\,0$

Load current, $\vec{I_R} = I_R(\cos \phi_R - j \sin \phi_R)$

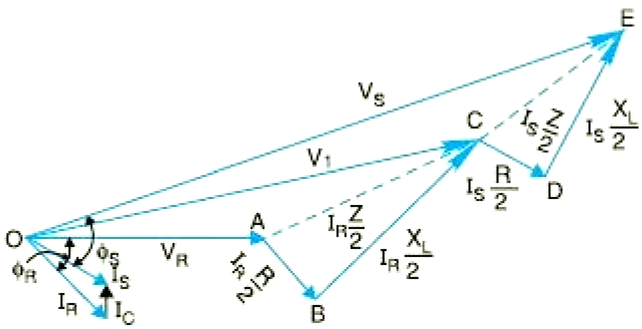


Fig. 12

Voltage across C, $\quad \vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2$

$$= V_R + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)$$

Capacitive current, $\quad \vec{I_C} = j\,\omega\,C\,\vec{V_1} = j\,2\pi f\,C\,\vec{V_1}$

Sending end current, $\quad \vec{I_S} = \vec{I_R} + \vec{I_C}$

Sending end voltage, $\quad \vec{V_S} = \vec{V_1} + \vec{I_S}\,\frac{\vec{Z}}{2} = \vec{V_1} + \vec{I_S}\left(\frac{R}{2} + j\frac{X_L}{2}\right)$

2.9 Nominal π Method

In this method, capacitance of each conductor (*i.e.*, line to neutral) is divided into two halves; one half being lumped at the sending end and the other half at the receiving end as shown in Fig. 13. It is obvious that capacitance at the sending end has no effect on the line drop. However, its charging current must be added to line current in order to obtain the total sending end current.

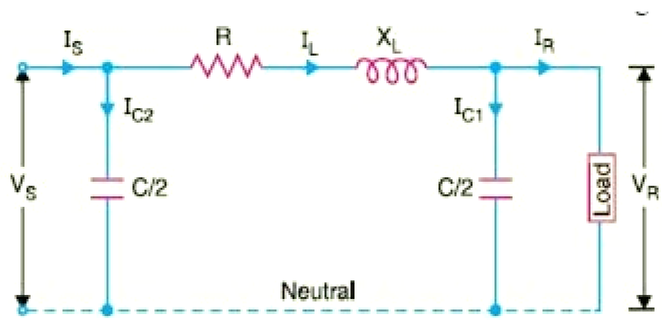


Fig. 13

Let $\quad I_R$ = load current per phase
R = resistance per phase
X_L = inductive reactance per phase
C = capacitance per phase
$\cos \phi_R$ = receiving end power factor (*lagging*)
V_S = sending end voltage per phase

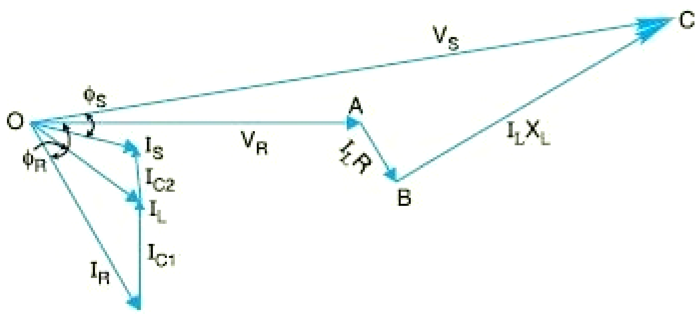
The *phasor diagram for the circuit is shown in Fig. 14. Taking the receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j\,0$$

Load current, $\quad \vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

Charging current at load end is

$$\vec{I_{C1}} = j\,\omega\,(C/2)\,\vec{V_R} = j\,\pi\,f\,C\,\vec{V_R}$$



Line current, $\quad \vec{I_L} = \vec{I_R} + \vec{I_{C1}}$

Sending end voltage, $\quad \vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \vec{I_L}\,(R + jX_L)$

Charging current at the sending end is

$$\vec{I_{C2}} = j\,\omega\,(C/2)\,\vec{V_S} = j\,\pi\,f\,C\,\vec{V_S}$$

$\therefore$ Sending end current, $\quad \vec{I_S} = \vec{I_L} + \vec{I_{C2}}$

2.10 Generalized Circuit Constants of a Transmission Line

In any four terminal network, the input voltage and input current can be expressed in terms of output voltage and output current. Incidentally, a transmission line is a 4-terminal network ; two input terminals where power enters the network and two output terminals where power leaves the network.

Therefore, the input voltage $\left(\overrightarrow{V_S}\right)$ and input current $\left(\overrightarrow{I_S}\right)$ of a 3-phase transmission line can be expressed as :

$$\overrightarrow{V_S} = \vec{A}\,\overrightarrow{V_R} + \vec{B}\,\overrightarrow{I_R}$$

$$\overrightarrow{I_S} = \vec{C}\,\overrightarrow{V_R} + \vec{D}\,\overrightarrow{I_R}$$

where $\overrightarrow{V_S}$ = sending end voltage per phase

$\overrightarrow{I_S}$ = sending end current

$\overrightarrow{V_R}$ = receiving end voltage per phase

$\overrightarrow{I_R}$ = receiving end current

and $\vec{A}, \vec{B}, \vec{C}$ *and* $\vec{D}$(generally complex numbers) are the constants known as *generalized circuit constants* of the transmission line. The values of these constants depend upon the particular method adopted for solving a transmission line. Once the values of these constants are known, performance calculations of the line can be easily worked out. The following points may be kept in mind:

i. The constants $\vec{A}, \vec{B}, \vec{C}\ and \vec{D}$ are generally complex numbers.

ii. The constants $\vec{A}\ and \vec{D}$ are dimensionless whereas the dimensions of $\vec{B}\ and\ \vec{C}$ are ohms and siemen respectively.

iii. For a given transmission line, $\vec{A} = \vec{D}$

iv. For a given transmission line, $\vec{A}\vec{D} - \vec{B}\vec{C} = 1$

We shall establish the correctness of above characteristics of generalized circuit constants in the following discussion.

2.11 Determination of Generalized Constants for Transmission Lines

As stated previously, the sending end voltage (V_S) and sending end current (I_S) of a transmission line can be expressed as:

$$\overrightarrow{V_S} = \vec{A}\overrightarrow{V_R} + \vec{B}\overrightarrow{I_R} \quad \text{(i)}$$

$$\overrightarrow{I_S} = \vec{C}\overrightarrow{V_R} + \vec{D}\overrightarrow{I_R} \quad \text{(ii)}$$

We shall now determine the values of these constants for different types of transmission lines.

1. Short lines:

In short transmission lines, the effect of line capacitance is neglected. Therefore, the line is considered to have series impedance. Fig. 15 shows the circuit of a 3-phase transmission line on a single phase basis.

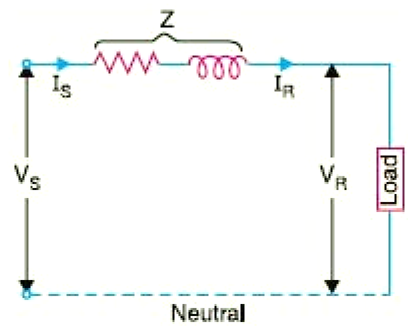


Fig. 15

Here $\overrightarrow{I_S} = \overrightarrow{I_R}$ (iii)

and $\overrightarrow{V_S} = \overrightarrow{V_R} + \overrightarrow{I_R}\, Z$ (iv)

Comparing these with eqs. (*i*) and (*ii*), we have,

$$\vec{A} = 1 ; \quad \vec{B} = \vec{Z}, \quad \vec{C} = 0 \quad \text{and} \quad \vec{D} = 1$$

Incidentally ; $\vec{A} = \vec{D}$

and $\vec{A}\vec{D} - \vec{B}\vec{C} = 1 \times 1 - \vec{Z} \times 0 = 1$

2. Medium lines – Nominal T method:

In this method, the whole line to neutral capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on either side as shown in Fig. 16.

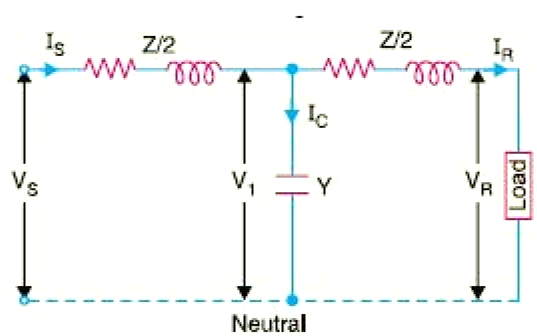


Fig. 16

Here $\vec{V_S} = \vec{V_1} + \vec{I_S}\vec{Z_S}/2$

and $\vec{V_1} = \vec{V_R} + \vec{I_R}\vec{Z_S}/2$

Now, $\vec{I_C} = \vec{I_S} - \vec{I_R}$

$$= \vec{V_1}\,\vec{Y} \quad \text{where } Y = \text{shunt admittance}$$

$$= \vec{Y}\left(\vec{V_R} + \frac{\vec{I_R}\,\vec{Z}}{2}\right)$$

$$\therefore \qquad \vec{I_S} = \vec{I_R} + \vec{Y}\,\vec{V_R} + \vec{Y}\,\frac{\vec{I_R}\,\vec{Z}}{2}$$

$$= \vec{Y}\,\vec{V_R} + \vec{I_R}\left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right) \qquad \ldots(vi)$$

Substituting the value of V_1 in eq. (v), we get,

$$\vec{V_S} = \vec{V_R} + \frac{\vec{I_R}\,\vec{Z}}{2} + \frac{\vec{I_S}\,\vec{Z}}{2}$$

Substituing the value of I_S, we get,

$$\vec{V_S} = \left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right)\vec{V_R} + \left(\vec{Z} + \frac{\vec{Y}\,\vec{Z}^2}{4}\right)\vec{I_R} \qquad \ldots(vii)$$

Comparing eqs. (vii) and (vi) with those of (i) and (ii), we have,

$$\vec{A} = \vec{D} = 1 + \frac{\vec{Y}\,\vec{Z}}{2}\ ; \quad \vec{B} = \vec{Z}\left(1 + \frac{\vec{Y}\,\vec{Z}}{4}\right)\ ; \ \vec{C} = \vec{Y}$$

Incidentally : $\quad \vec{A}\,\vec{D} - \vec{B}\,\vec{C} = \left(1 + \frac{Y\,Z}{2}\right)^2 - Z\left(1 + \frac{Y\,Z}{4}\right)Y$

$$= 1 + \frac{Y^2 Z^2}{4} + Y\,Z - Z\,Y - \frac{Z^2 Y^2}{4} = 1$$

3. Medium lines—Nominal π method:

In this method, line-to-neutral capacitance is divided into two halves ; one half being concentrated at the load end and the other half at the sending end as shown in Fig. 25.

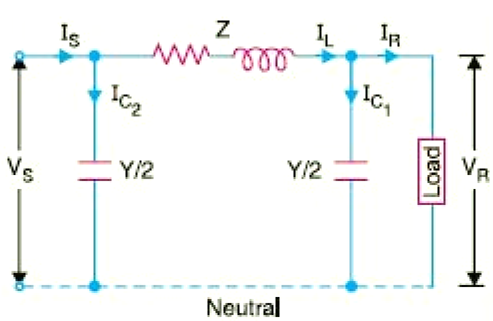


Fig. 17

Here, $\vec{Z} = R + jX_L$ = series impedenace/phase

$\vec{Y} = j\,\omega\,C$ = shunt admittance

$\vec{I_S} = \vec{I_L} + \vec{I_{C2}}$

or $\vec{I_S} = \vec{I_L} + \vec{V_S}\,\vec{Y}/2$ ...(*viii*)

Also $$\vec{I_L} = \vec{I_R} + \vec{I_{C1}} = \vec{I_R} + \vec{V_R}\,\vec{Y}/2 \qquad ...(ix)$$

Now $$\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \left(\vec{I_R} + \vec{V_R}\,\vec{Y}/2\right)\vec{Z} \text{ (Putting the value of } \vec{I_L}\text{)}$$

$\therefore$ $$\vec{V_S} = \vec{V_R}\left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right) + \vec{I_R}\,\vec{Z} \qquad ...(x)$$

Also $$\vec{I_S} = \vec{I_L} + \vec{V_S}\,\vec{Y}/2 = \left(\vec{I_R} + \vec{V_R}\,\vec{Y}/2\right) + \vec{V_S}\,\vec{Y}/2$$

(Putting the value of $\vec{I_L}$)

Putting the value of $\vec{V_S}$ from eq. (*x*), we get,

$$\vec{I_S} = \vec{I_R} + \vec{V_R}\frac{\vec{Y}}{2} + \frac{\vec{Y}}{2}\left\{\vec{V_R}\left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right) + \vec{I_R}\,\vec{Z}\right\}$$

$$= \vec{I_R} + \vec{V_R}\frac{\vec{Y}}{2} + \frac{\vec{V_R}\,\vec{Y}}{2} + \frac{\vec{V_R}\,\vec{Y}^2\,\vec{Z}}{4} + \frac{\vec{Y}\,\vec{I_R}\,\vec{Z}}{2}$$

$$= \vec{I_R}\left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right) + \vec{V_R}\,\vec{Y}\left(1 + \frac{\vec{Y}\,\vec{Z}}{4}\right) \qquad ...(xi)$$

Comparing equations (*x*) and (*xi*) with those of (*i*) and (*ii*), we get,

$$\vec{A} = \vec{D} = \left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right);\ \vec{B} = \vec{Z};\ \vec{C} = \vec{Y}\left(1 + \frac{\vec{Y}\,\vec{Z}}{4}\right)$$

Also $$\vec{A}\,\vec{D} - \vec{B}\,\vec{C} = \left(1 + \frac{Y\,Z}{2}\right)^2 - Z\,Y\left(1 + \frac{Y\,Z}{4}\right)$$

$$= 1 + \frac{Y^2Z^2}{4} + Y\,Z - Z\,Y - \frac{Z^2Y^2}{4} = 1$$

PS-II: UNIT-IV

TRANSMISSION LINE PARAMETERS

Learning objectives:

- Able to understand the performance analysis of long transmission lines.

Syllabus:

Long Transmission Line-Rigorous Solution, evaluation of A,B,C,D Constants, Interpretation of the Long Line Equations, Incident, Reflected and Refracted Waves -Surge Impedance and SIL of Long Lines, Wave Length and Velocity of Propagation of Waves - Representation of Long Lines - Equivalent-T and Equivalent Pie network models (numerical problems).

Learning outcomes:

Students will be able to

- Sketch the equivalent representation of long lines.
- Derive a rigorous solution for long transmission lines.
- Evaluate A,B,C,D constants.
- Explain Surge impedance and surge impedance loading.

UNIT-IV

TRANSMISSION LINE PARAMETERS

Long Line Model

For accurate modeling of the transmission line we must not assume that the parameters are lumped but are distributed throughout line. The single-line diagram of a long transmission line is shown in Fig. 4.1. The length of the line is l. Let us consider a small strip Δx that is at a distance x from the receiving end. The voltage and current at the end of the strip are V and I respectively and the beginning of the strip are V + ΔV and I + Δ I respectively. The voltage drop across the strip is then ΔV. Since the length of the strip is Δx , the series impedance and shunt admittance are z Δx and y Δx . It is to be noted here that the total impedance and admittance of the line are

$$Z = z \times l \text{ and } Y = y \times l \tag{4.1}$$

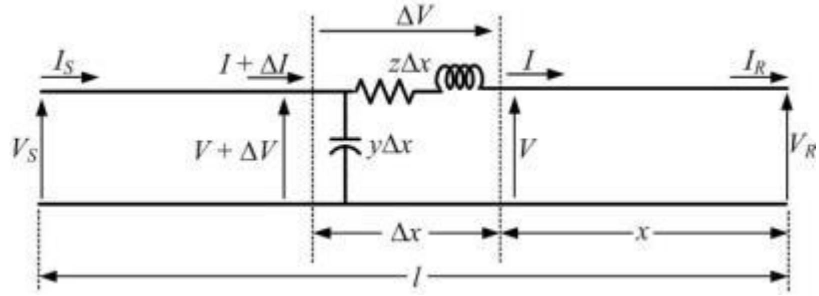


Fig. 4.1 Long transmission line representation.

From the circuit of Fig. 4.1 we see that

$$\Delta V = Iz\,\Delta x \Rightarrow \frac{\Delta V}{\Delta x} = Iz \tag{4.2}$$

Again as $\Delta x \to 0$, from (4.2) we get

$$\frac{dV}{dx} = Iz \tag{4.3}$$

Now for the current through the strip, applying KCL we get

$$\Delta I = (V + \Delta V)y\,\Delta x = Vy\,\Delta x + \Delta Vy\,\Delta x \tag{4.4}$$

The second term of the above equation is the product of two small quantities and therefore can be neglected. For $\Delta x \to 0$ we then have

$$\frac{dI}{dx} = Vy \tag{4.5}$$

Taking derivative with respect to x of both sides of (4.3) we get

$$\frac{d}{dx}\left(\frac{dV}{dx}\right) = z\frac{dI}{dx}$$

Substitution of (4.5) in the above equation results

$$\frac{d^2V}{dx^2} - yzV = 0 \tag{4.6}$$

The roots of the above equation are located at ±√(yz). Hence the solution of (4.6) is of the form

$$V = A_1 e^{x\sqrt{yz}} + A_2 e^{-x\sqrt{yz}} \tag{4.7}$$

Taking derivative of (4.7) with respect to x we get

$$\frac{dV}{dx} = A_1\sqrt{yz}\,e^{x\sqrt{yz}} - A_2\sqrt{yz}\,e^{-x\sqrt{yz}} \tag{4.8}$$

Combining (4.3) with (4.8) we have

$$I = \frac{1}{z}\left(\frac{dV}{dx}\right) = \frac{A_1}{\sqrt{z/y}}e^{x\sqrt{yz}} - \frac{A_2}{\sqrt{z/y}}e^{-x\sqrt{yz}} \tag{4.9}$$

Let us define the following two quantities

$$Z_C = \sqrt{\frac{z}{y}}\ \Omega \text{ which is called the } \textbf{\textit{characteristic impedance}} \tag{4.10}$$

$$\gamma = \sqrt{yz} \text{ which is called the } \textbf{\textit{propagation constant}} \tag{4.11}$$

Then (4.7) and (4.9) can be written in terms of the characteristic impedance and propagation constant as

$$V = A_1 e^{\gamma x} + A_2 e^{-\gamma x} \tag{4.12}$$

$$I = \frac{A_1}{Z_C}e^{\gamma x} - \frac{A_2}{Z_C}e^{-\gamma x} \tag{4.13}$$

Let us assume that x = 0. Then V = VR and I = IR . From (4.12) and (4.13) we then get

$$V_R = A_1 + A_2 \tag{4.14}$$

$$I_R = \frac{A_1}{Z_C} - \frac{A_2}{Z_C} \tag{4.15}$$

Solving (4.14) and (4.15) we get the following values for A1 andA2 .

$$A_1 = \frac{V_R + Z_C I_R}{2} \text{ and } A_2 = \frac{V_R - Z_C I_R}{2}$$

Also note that for x = l we have V = V_s and I = I_S . Therefore replacing x by l and substituting the values of A_1 and A_2 in (4.12) and (4.13) we get

$$V_S = \frac{V_R + Z_C I_R}{2} e^{\gamma l} + \frac{V_R - Z_C I_R}{2} e^{-\gamma l} \tag{4.16}$$

$$I_S = \frac{V_R/Z_C + I_R}{2} e^{\gamma l} - \frac{V_R/Z_C - I_R}{2} e^{-\gamma l} \tag{4.17}$$

Noting that

$$\frac{e^{\gamma l} - e^{-\gamma l}}{2} = \sinh \gamma l \text{ and } \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \cosh \gamma l$$

We can rewrite (4.16) and (4.17) as

$$V_S = V_R \cosh \gamma l + Z_C I_R \sinh \gamma l \tag{4.18}$$

$$I_S = V_R \frac{\sinh \gamma l}{Z_C} + I_R \cosh \gamma l \tag{4.19}$$

The ABCD parameters of the long transmission line can then be written as

$$A = D = \cosh \gamma l \tag{4.20}$$

$$B = Z_C \sinh \gamma l \tag{4.21}$$

$$C = \frac{\sinh \gamma l}{Z_C} \tag{4.22}$$

Evaluation of ABCD Constants

The ABCD constants of a long line can be evaluated from the results given above. It must be noted that $\gamma = \sqrt{yz}$ is in general a complex number and can be expressed as

$$\gamma = \alpha + j\beta$$

The hyperbolic function of complex numbers involved in evaluating ABCD constants can be computed by any one of the three methods given below.

Method I

cosh (αl + jβl) = cosh αl cos βl+ j sinh αl sin βl
sinh (αl + jβl) = sinh αl cos βl + j cosh αl sin βl

Method II

$$Cosh\,\gamma l = 1 + \frac{\gamma^2 l^2}{2!} + \frac{\gamma^4 l^4}{4!} + \,\dots\dots. \approx \mathbf{1} + \frac{\boldsymbol{YZ}}{\mathbf{2}}$$
$$sinh\,\gamma l = \gamma l + \frac{\gamma^3 l^3}{3!} + \frac{\gamma^5 l^5}{5!} + \,\dots\dots. \approx \sqrt{\boldsymbol{YZ}}\left(\mathbf{1} + \frac{\boldsymbol{YZ}}{\mathbf{2}}\right)$$

Method III

$$Cosh\,(\alpha l + j\beta l) = \frac{e^{\alpha l}e^{j\beta l} + e^{-\alpha l}e^{-j\beta l}}{2} = \frac{1}{2}\left(e^{\alpha l}\angle\beta l + e^{-\alpha l}\angle-\beta l\right)$$
$$sinh\,(\alpha l + j\beta l) = \frac{e^{\alpha l}e^{j\beta l} - e^{-\alpha l}e^{-j\beta l}}{2} = \frac{1}{2}\left(e^{\alpha l}\angle\beta l - e^{-\alpha l}\angle-\beta l\right)$$

Equivalent- π Representation of a Long Line

The π -equivalent of a long transmission line is shown in Fig. 4.2. In this the series impedance is denoted by Z' while the shunt admittance is denoted by Y' , the ABCD parameters are defined as

$$A = D = \left(\frac{Y'Z'}{2} + 1\right) \tag{4.23}$$

$$B = Z'\Omega \tag{4.24}$$

$$C = Y'\left(\frac{Y'Z'}{4} + 1\right)\text{mho} \tag{4.25}$$

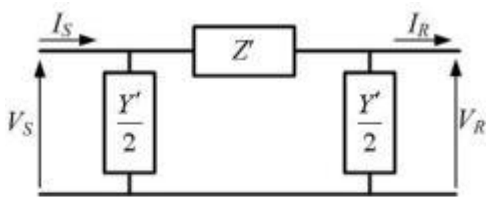


Fig. 4.2 Equivalent pie representation of a long transmission line.

Comparing (4.21) with (4.24) we can write

$$Z' = Z_C \sinh \gamma l = \sqrt{\frac{z}{y}} \sinh \gamma l = zl \frac{\sinh \gamma l}{l\sqrt{yz}} = Z \frac{\sinh \gamma l}{\gamma l} \quad \Omega \tag{4.26}$$

where $Z = zl$ is the total impedance of the line. Again comparing (4.20) with (4.23) we get

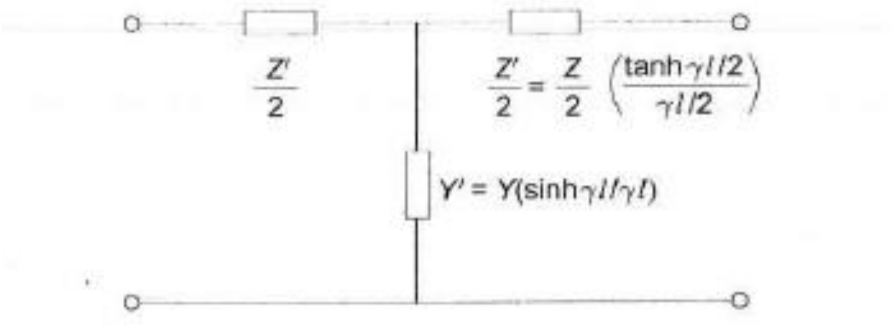
$$\cosh \gamma l = \frac{Y'Z'}{2} + 1 = \frac{Y'}{2} Z_C \sinh \gamma l + 1 \tag{4.27}$$

Rearranging (4.27) we get

$$\begin{aligned} \frac{Y'}{2} &= \frac{1}{Z_C} \frac{\cosh \gamma l - 1}{\sinh \gamma l} = \frac{1}{Z_C} \tanh(\gamma l/2) = \sqrt{\frac{y}{z}} \tanh(\gamma l/2) = \frac{yl}{2} \frac{\tanh(\gamma l/2)}{(l/2)\sqrt{yz}} \\ &= \frac{Y}{2} \frac{\tanh(\gamma l/2)}{(\gamma l/2)} \end{aligned} \tag{4.28}$$

Where $Y = yl$ is the total admittance of the line. Note that for small values of l, $\sinh \gamma l = \gamma l$ and $\tanh(\gamma l/2) = \gamma l/2$. Therefore from (4.26) we get $Z = Z'$ and from (4.28) we get $Y = Y'$. This implies that when the length of the line is small, the nominal- pie representation with lumped parameters is fairly accurate. However the lumped parameter representation becomes erroneous as the length of the line increases.

Equivalent-T network parameters of a transmission line are obtained on similar lines. The equivalent-T network is shown in figure.



Equivalent-T network of a transmission line

INTERPRETATION OF THE LONG LINE EOUATIONS

As already said, γ is a complex number which can be expressed as

$$\gamma = \alpha + j\beta$$

The real part α is called the attenuation constant and the imaginary part β is called the phase constant. Now V, of Eq. (4.16) can be written as

$$V_x = \left|\frac{V_R + Z_c I_R}{2}\right| e^{\alpha x} e^{j(\beta x + \varphi_1)} + \left|\frac{V_R - Z_c I_R}{2}\right| e^{-\alpha x} e^{-j(\beta x - \varphi_2)} \tag{4.29}$$

Where

$$\varphi_1 = \angle\,(V_R + Z_c I_R)$$

$$\varphi_2 = \angle\,(V_R - Z_c I_R)$$

The instantaneous voltage $v_x(t)$ can be written from equation as

$$V_x(t) = Re\left[\sqrt{2}\left|\frac{V_R + Z_c I_R}{2}\right| e^{\alpha x} e^{j(\omega t + \beta x + \varphi_1)} + \sqrt{2}\left|\frac{V_R - Z_c I_R}{2}\right| e^{-\alpha x} e^{-j(\omega t + \beta x - \varphi_2)}\right] \tag{4.30}$$

The instantaneous voltage consists of two terms each of which is a function of two variables- time and distance. Thus they represent two travelling waves,i.e.,

$$V_x = V_{x1} + V_{x2} \tag{4.31}$$

$$V_{x1} = \sqrt{2}\left|\frac{V_R + Z_c I_R}{2}\right| e^{\alpha x}\, cos(\omega t + \beta x + \varphi_1) \tag{4.32}$$

At any instant of time t, v_{x1} is sinusoidally distributed along the distance from the receiving-end with amplitude increasing exponentially with distance, as shown in Fig. 4.3 ($\alpha > 0$ for a line having resistance).

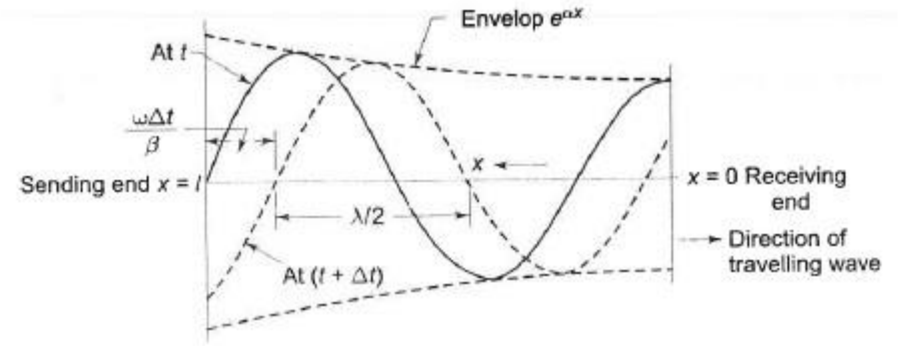


Fig. 4.2 Incident Wave

After time Δt, the distribution advances in distance phase by (ωΔt/β). Thus this wave is travelling towards the receiving-end and is the incident wave. Line losses cause its amplitude to decrease exponentially in going from the sending to the receiving-end.

Now

$$V_{x2} = \sqrt{2}\left|\frac{V_R - Z_c I_R}{2}\right| e^{-\alpha x}\, cos(\omega t - \beta x + \varphi_2) \tag{4.33}$$

After time Δt the voltage distribution retards in distance phase by (ωΔt/β). This is the reflected wave travelling from the receiving-end to the sending-end with amplitude decreasing exponentially in going from the receiving-end to the sending-end, as shown in Fig 4.3

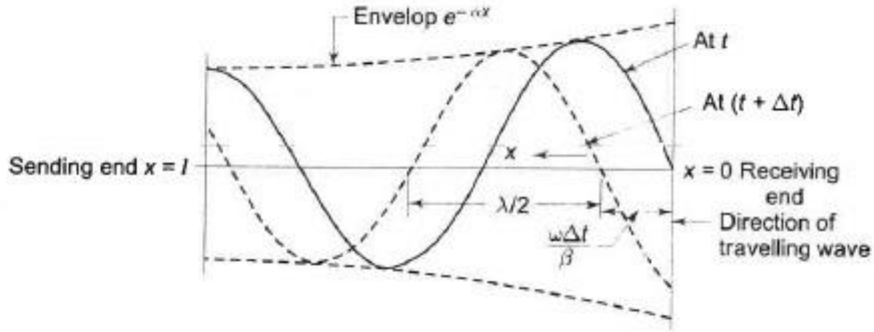


Fig. 4.3 Reflected wave

At any point along the line, the voltage is the sum of incident and reflected voltage waves present at the point [Eq. (4.31)]. The same is true of current waves. Expressions for incident and reflected current waves can be similarly written down by proceeding from Eq. (4.17).If Z_c is pure resistance, current waves can be simply obtained from voltage waves by dividing by Z_c.

If the load impedance $Z_L= V_R/I_R = Z_c$, i.e. the line is terminated in its characteristic impedance, the reflected voltage wave is zero ($V_R – Z_c I_R = 0$).

A line terminated in its characteristic impedance is called the infinite line. The incident wave under this condition cannot distinguish between a termination and an infinite continuation of the line.

Power system engineers normally caII Z_c the surge impedance. It has a value of about 400 ohms for an overhead line and its phase angle normally varies from 0^o to $– 15^o$. For underground cables Z_c is roughly one-tenth of the value for overhead lines. The term surge impedance is, however, used in connection with surges(due to lightning or switching) or transmission lines, Where the lines loss can be neglected such that

$Z_c = Z_s = (j\omega L/-j\omega C)^{1/2} = (L/C)^{1/2}$, a pure resistance.

Surge Impedance Loading (SIL) of a transmission line is defined as the power delivered by a line to purely resistive load equal in value to the surge impedance of the line. Thus for a line having 400 ohms surge impedance,

$$SIL = \sqrt{3}\frac{|V_R|}{\sqrt{3} \times 400}|V_R| \times 1000\, KW$$

$$SIL = 2.5\,|V_R|^2\, KW$$

where $|V_R|$ is the line-to-line receiving-end voltage in kV. Sometimes, it is found convenient to express line loading in per unit of SIL, i.e. as the ratio of the power transmitted to surge impedance loading.

At any time the voltage and current vary harmonically along the line with respect to x, the space coordinate. A complete voltage or current cycle along the line corresponds to a change of 2π rad in the angular argument βx. The corresponding line length is defined as the wavelength.

If β is expressed in rad/m,

$\lambda = 2\pi/\beta$ m

Now for a typical power transmission line 'g' (shunt conductance/unit length) = 0

$r \ll \omega L$

$\gamma = (yz)^{1/2} = (j\omega c\,(r + j\omega L))^{1/2}$

$= j\omega\,(LC)^{1/2}\,(1 - j\,r/\omega L)^{1/2}$

Or

$\gamma = \alpha + j\beta = j\omega (LC)^{1/2}\,(1 - j\,r/\omega L)$

$\alpha = r/2 \times (C/L)^{1/2}$

$\beta = \omega\,(LC)^{1/2}$

Now time for a phase change of 2π is 1/f s, where $f = \omega/2\pi$ is the frequency in cycles/s. During this time the wave travels a distance equal to λ. i.e. one wavelength.

Velocity of propagation of wave, $v = \lambda/(1/f) = \lambda f$ m/s

which is a well known result.

For a lossless transmission line (R = 0, G = 0),

$\gamma = (yz)^{1/2} = j\omega\,(LC)^{1/2}$

such that $\alpha = 0$, $\beta = \omega (LC)^{1/2}$

$\lambda = 2\pi/\beta = 2\pi/(\omega (LC)^{1/2}) = 1/(f\,(LC)^{1/2})$ m

and

$v = f\lambda = 1/(LC)^{1/2}$ m/s

For a single-phase transmission line

$$L = \frac{\mu_o}{2\pi} ln \frac{D}{r'}$$

$$C = \frac{2\pi k_o}{\ln\left(\frac{D}{r}\right)}$$

$$v = \frac{1}{\left(\frac{\mu_o}{2\pi} ln\frac{D}{r'} \frac{2\pi k_o}{\ln\left(\frac{D}{r}\right)}\right)^{1/2}}$$

Since r and r' are quite close to each other, when log is taken, it is sufficiently accurate to assume that ln(D/r') = ln(D/r).

$$v = \frac{1}{(\mu_o k_o)^{1/2}} = velocity\ of\ light$$

The actual velocity of the propagation of wave along the line would be somewhat less than the velocity of light.

Wave length of a 50 Hz power transmission is approximately given by

$\lambda = 3\times10^8 /50 = 6000$ km

Characterization of a Long Lossless Line

For a lossless line, the line resistance is assumed to be zero. The characteristic impedance then becomes a pure real number and it is often referred to as the surge impedance**.** The propagation constant becomes a pure imaginary number. Defining the propagation constant as $\gamma = j\beta$ and replacing l by x we can rewrite (4.18) and (4.19) as

$$V = V_R \cos\beta x + jZ_C I_R \sin\beta x \tag{4.34}$$

$$I = jV_R \frac{\sin\beta x}{Z_C} + I_R \cos\beta x \tag{4.35}$$

The term surge impedance loading or *SIL* is often used to indicate the nominal capacity of the line. The surge impedance is the ratio of voltage and current at any point along an infinitely long line. The term *SIL* or natural power is a measure of power delivered by a transmission line when terminated by surge impedance and is given by

$$SIL = P_n = \frac{V_0^2}{Z_C} \tag{4.36}$$

Where V_0 is the rated voltage of the line.

At *SIL* $Z_C = V_R / I_R$ and hence from equations (4.34) and (4.35) we get

$$V = V_R e^{\gamma x} = V_R e^{-j\beta x} \tag{4.37}$$

$$I = I_R e^{\gamma x} = I_R e^{-j\beta x} \tag{4.38}$$

This implies that as the distance x changes, the magnitudes of the voltage and current in the above equations do not change. The voltage then has a flat profile all along the line. Also as Z_C is real, V and I are in phase with each other all throughout the line. The phase angle difference between the sending end voltage and the receiving end voltage is then $\theta = \beta l$. This is shown in Fig. 4.4.

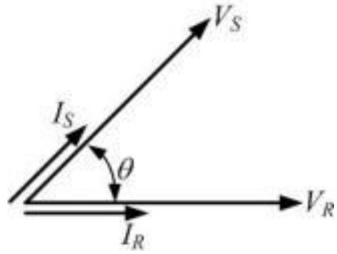


Fig. 4.4 Voltage-current relationship in naturally loaded line.

Voltage and Current Characteristics

For the analysis presented below we assume that the magnitudes of the voltages at the two ends are the same. The sending and receiving voltages are given by

$$V_S = |V_S|\angle\delta \quad \text{and} \quad V_R = |V_R|\angle 0^\circ \tag{4.39}$$

where δ is angle between the sources and is usually called the load angle . As the total length of the line is l , we replace x by l to obtain the sending end voltage from (4.16) as

$$V_S = |V_S|\angle\delta = \frac{|V_R| + Z_C I_R}{2} e^{j\theta} + \frac{|V_R| - Z_C I_R}{2} e^{-j\theta} = |V_R|\cos\theta + jZ_C I_R \sin\theta \tag{4.40}$$

Solving the above equation we get

$$I_R = \frac{|V_S|\angle\delta - |V_R|\cos\theta}{jZ_C \sin\theta} \tag{4.41}$$

Substituting (4.41) in (4.34), the voltage equation at a point in the transmission line that is at a distance x from the receiving end is obtained as

$$V = |V_R|\cos\beta x + \frac{|V_S|\angle\delta - |V_R|\cos\theta}{jZ_C \sin\theta}\left(jZ_C \sin\beta x\right) = \frac{|V_S|\angle\delta \sin\beta x + |V_R|\sin(\theta - \beta x)}{\sin\theta} \tag{4.42}$$

In a similar way the current at that point is given by

$$I = \frac{-j}{Z_C}\left[\frac{|V_S|\angle\delta\cos\beta x - |V_R|\cos(\theta - \beta x)}{\sin\theta}\right] \tag{4.43}$$

When the system is unloaded, the receiving end current is zero ($I_R = 0$). Therefore we can rewrite (4.40) as

$$V_S = |V_S| \angle \delta = |V_R| \cos\theta \tag{4.44}$$

Mid Point Voltage and Current of Loaded Lines

The midpoint voltage of a transmission line is of significance for the reactive compensation of transmission lines. To obtain an expression of the midpoint voltage, let us assume that the line is loaded (i.e., the load angle *d* is not equal to zero). At the midpoint of the line we have $x = l/2$ such that $\beta x = q/2$. Let us denote the midpoint voltage by V_M. Let us also assume that the line is symmetric, i.e., $|V_S| = |V_R| = V$. We can then rewrite equation (4.40) to obtain

$$V_M = \frac{(V\angle\delta + V)\sin(\theta/2)}{\sin\theta}$$

Again noting that

$$\begin{aligned} V\angle\delta + V &= V(\cos\delta + j\sin\delta + 1) \\ &= V\sqrt{2 + 2\cos\delta}\,\tan^{-1}\left(\frac{\sin\delta}{1+\cos\delta}\right) = 2V\cos(\delta/2)\angle(\delta/2) \end{aligned}$$

We obtain the following expression of the midpoint voltage

$$V_M = \frac{V\cos(\delta/2)}{\cos(\theta/2)}\angle\left(\delta/_2\right) \tag{4.45}$$

The midpoint current is similarly given by

$$I_M = \frac{-j}{Z_C}\left[\frac{(V\angle\delta - V)\cos(\theta/2)}{\sin\theta}\right] = \frac{V}{Z_C}\frac{\sin(\delta/2)}{\sin(\theta/2)}\angle\left(\delta/_2\right) \tag{4.46}$$

The phase angle of the mid point voltage is half the load angle always. Also the mid point voltage and current are in phase, i.e., the power factor at this point is unity. The variation in the magnitude of voltage with changes in load angle is maximum at the mid point. The voltage at this point decreases with the increase in δ . Also as the power through a lossless line is constant through out its length and the mid point power factor is unity, the mid point current increases with an increase in δ.

Power in a Lossless Line

The power flow through a lossless line can be given by the mid point voltage and current equations given in (4.42) and (4.43). Since the power factor at this point is unity, real power over the line is given by

$$P_e = V_M I_M^* = \frac{V^2}{Z_C \sin\theta}\sin\delta \tag{4.47}$$

If $V = V_0$, the rated voltage, we can rewrite the above expression in terms of the natural power as

$$P_e = \frac{P_n}{\sin\theta}\sin\delta \tag{4.48}$$

For a short transmission line we have

$$Z_C \sin\theta \cong Z_C\theta = \left(\sqrt{l/c}\right)\left(\omega\sqrt{lc}\right)\tau = \omega l \tau = X \tag{4.49}$$

Where X is the total reactance of the line. Equation (4.47) then can be modified to obtain the well known power transfer relation for the short line approximation as

$$P_e = \frac{V^2}{X}\sin\delta \tag{4.50}$$

In general it is not necessary for the magnitudes of the sending and receiving end voltages to be same. The power transfer relation given in (4.48) will not be valid in that case. To derive a general expression for power transfer, we assume

$V_S = |V_S|\angle\delta$ and $V_R = |V_R|\angle 0^\circ$

If the receiving end real and reactive powers are denoted by P_R and Q_R respectively, we can write

$$V_S = |V_S|(\cos\delta + j\sin\delta) = |V_R|\cos\theta + jZ_C\frac{P_R - jQ_R}{|V_R|}\sin\theta$$

Equating real and imaginary parts of the above equation we get

$$|V_S|\cos\delta = |V_R|\cos\theta + \frac{Z_C Q_R}{|V_R|}\sin\theta \tag{4.51}$$

and

$$|V_S|\sin\delta = \frac{Z_C P_R}{|V_R|}\sin\theta \tag{4.52}$$

Rearranging (4.52) we get the power flow equation for a lossless line as

$$P_e = P_S = P_R = \frac{|V_S||V_R|}{Z_C\sin\theta}\sin\delta \tag{4.53}$$

To derive expressions for the reactive powers, we rearrange (4.51) to obtain the reactive power delivered to the receiving end as

$$Q_R = \frac{|V_S||V_R|\cos\delta - |V_R|^2\cos\theta}{Z_C\sin\theta} \tag{4.54}$$

Again from equation (4.43) we can write

$$I_S = \frac{-j}{Z_C}\left[\frac{|V_S|\cos\theta\angle\delta - |V_R|}{\sin\theta}\right] \tag{4.55}$$

The sending end apparent power is then given by

$$P_S + jQ_S = V_S I_S^* = |V_S|\angle\delta \frac{j}{Z_C}\left[\frac{|V_S|\cos\theta\angle-\delta - |V_R|}{\sin\theta}\right] = j\frac{|V_S|^2\cos\theta}{Z_C\sin\theta} - j\frac{|V_S||V_R|\angle\delta}{Z_C\sin\theta}$$

Equating the imaginary parts of the above equation we get the following expression for the reactive generated by the source

$$Q_S = \frac{|V_S|^2\cos\theta - |V_S||V_R|\cos\delta}{Z_C\sin\theta} \tag{4.56}$$

The reactive power absorbed by the line is then

$$Q_L = Q_S - Q_R = \frac{\left(|V_S|^2 + |V_R|^2\right)\cos\theta - 2|V_S||V_R|\cos\delta}{Z_C\sin\theta} \tag{4.57}$$

It is important to note that if the magnitude of the voltage at the two ends is equal, i.e., $|V_S| = |V_R| = V$, the reactive powers at the two ends become negative of each other, i.e., $Q_S = Q_R$. The net reactive power absorbed by the line then becomes twice the sending end reactive power, i.e., $Q_L = 2Q_S$. Furthermore, since $\cos\theta \approx 1$ for small values of θ, the reactive powers at the two ends for a short transmission line are given by

$$Q_S = \frac{V^2\cos\theta - V^2\cos\delta}{Z_C\sin\theta} \approx \frac{V^2}{X}(1-\cos\delta) = -Q_R \tag{4.58}$$

The reactive power absorbed by the line under this condition is given by

$$Q_L = \frac{2V^2}{X}(1-\cos\delta) \tag{4.59}$$

Substituting the above equation in (4.34) and (4.35) we get the voltage and current for the unloaded system as

$$V = \frac{|V_S|\angle\delta}{\cos\theta}\cos\beta x \tag{4.60}$$

UNIT-V

Over Head Line Insulators and Corona

Insulators

The overhead line conductors should be supported on the poles or towers in such a way that currents from conductors do not flow to earth through supports *i.e.*, line conductors must be properly insulated from supports. This is achieved by securing line conductors to supports with the help of *insulators*. The insulators provide necessary insulation between line conductors and supports and thus prevent any leakage current from conductors to earth. In general, the insulators should have the following desirable properties:

1. High mechanical strength in order to withstand conductor load, wind load etc.
2. High electrical resistance of insulator material in order to avoid leakage currents to earth.
3. High relative permittivity of insulator material in order that dielectric strength is high.
4. The insulator material should be non-porous; free from impurities and cracks otherwise the permittivity will be lowered.
5. High ratio of puncture strength to flashover.

The most commonly used material for insulators of overhead line is *porcelain* but glass, steatite and special composition materials are also used to a limited extent. Porcelain is produced by firing at a high temperature a mixture of kaolin, feldspar and quartz. It is stronger mechanically than glass, gives less trouble from leakage and is less affected by changes of temperature.

Types of Insulators

The successful operation of an overhead line depends to a considerable extent upon the proper selection of insulators. There are several types of insulators but the most commonly used are pin type, suspension type, strain insulator and shackle insulator.

Pin type insulators:

The part section of a pin type insulator is shown in Fig. 5. As the name suggests, the pin type insulator is secured to the cross-arm on the pole. There is a groove on the upper end of the insulator for housing the conductor. The conductor passes through this groove and is bound by the annealed wire of the same material as the conductor [See Fig. 5].

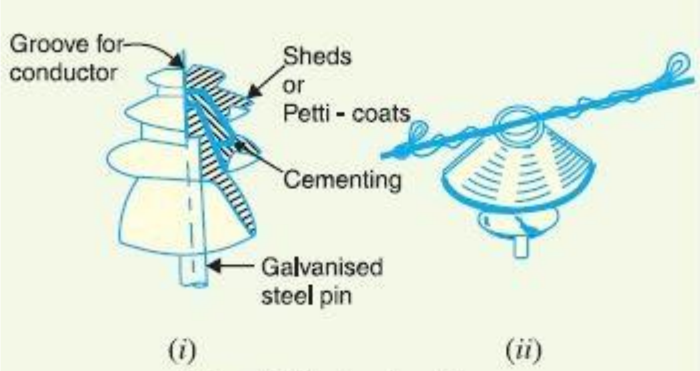


Fig. 5. Pin-type insulator

Pin type insulators are used for transmission and distribution of electric power at voltages up to 33 kV. Beyond operating voltage of 33 kV, the pin type insulators become too bulky and hence uneconomical.

Causes of insulator failure:

Insulators are required to withstand both mechanical and electrical stresses. The latter type is primarily due to line voltage and may cause the breakdown of the insulator. The electrical breakdown of the insulator can occur either by *flash-over* or *puncture.* In flashover, an arc occurs between the line conductor and insulator pin (*i.e.*, earth) and the discharge jumps across the *air gaps, following shortest distance. Fig. 6 shows the arcing distance (*i.e.* $a + b + c$) for the insulator. In case of flash-over, the insulator will continue to act in its proper capacity unless extreme heat produced by the arc destroys the insulator.

In case of puncture, the discharge occurs from conductor to pin through the body of the insulator. When such breakdown is involved, the insulator is permanently destroyed due to excessive heat. In practice, sufficient thickness of porcelain is provided in the insulator to avoid puncture by the line voltage. The ratio of puncture strength to flashover voltage is known as safety factor *i.e.*,

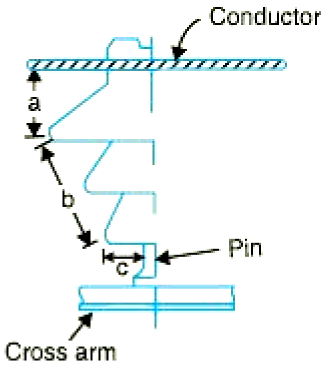


Fig. 6

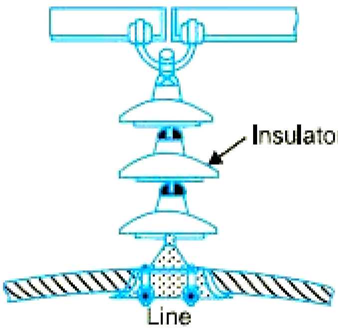


Fig. 7

$$\text{Safety factor of insulator} = \frac{\text{Puncture strength}}{\text{Flash} - \text{over voltage}}$$


Pin type insulator


Suspension insulator

It is desirable that the value of safety factor is high so that flash-over takes place before the insulator gets punctured. For pin type insulators, the value of safety factor is about 10.

Suspension type insulators:

The cost of pin type insulator increases rapidly as the working voltage is increased. Therefore, this type of insulator is not economical beyond 33 kV. For high voltages (>33 kV), it is a usual practice to use suspension type insulators shown in Fig. 7. They consist of a number of porcelain discs connected in series by metal links in the form of a string. The conductor is suspended at the bottom end of this string while the other end of the string is secured to the cross-arm of the tower. Each unit or disc is designed for low voltage, say 11 kV. The number of discs in series would obviously depend upon the working voltage. For instance, if the working voltage is 66 kV, then six discs in series will be provided on the string.

The insulator is generally dry and its surfaces have proper insulating properties. Therefore, arc can only occur through air gap between conductor and insulator pin.

Advantages

1. Suspension type insulators are cheaper than pin type insulators for voltages beyond 33 kV.
2. Each unit or disc of suspension type insulator is designed for low voltage, usually 11 kV. Depending upon the working voltage, the desired number of discs can be connected in series.
3. If anyone disc is damaged, the whole string does not become useless because the damaged disc can be replaced by the sound one.
4. The suspension arrangement provides greater flexibility to the line. The connection at the cross arm is such that insulator string is free to swing in any direction and can take up the position where mechanical stresses are minimum.
5. In case of increased demand on the transmission line, it is found more satisfactory to supply the greater demand by raising the line voltage than to provide another set of conductors. The additional insulation required for the raised voltage can be easily obtained in the suspension arrangement by adding the desired number of discs.
6. The suspension type insulators are generally used with steel towers. As the conductors run below the earthed cross-arm of the tower, therefore, this arrangement provides partial protection from lightning.

Strain insulators:

When there is a dead end of the line or there is corner or sharp curve, the line is subjected to greater tension. In order to relieve the line of excessive tension, strain insulators are used. For low voltage lines (< 11 kV), shackle insulators are used as strain insulators. However, for high voltage transmission lines, strain insulator consists of an assembly of suspension insulators as shown in Fig. 8. The discs of strain insulators are used in the vertical plane. When the tension in lines is exceedingly high, as at long river spans, two or more strings are used in parallel.

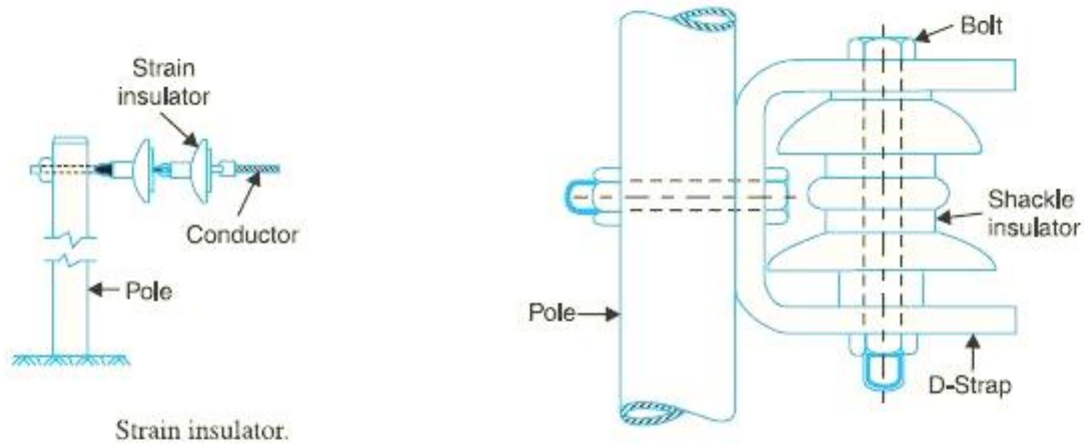


Fig. 8 Strain Insulator & Shackle Insulator

Shackle insulators:

In early days, the shackle insulators were used as strain insulators. But now a days, they are frequently used for low voltage distribution lines. Such insulators can be used either in a horizontal position or in a vertical position. They can be directly fixed to the pole with a bolt or to the cross arm. Fig. 8 shows a shackle insulator fixed to the pole. The conductor in the groove is fixed with a soft binding wire.

Potential Distribution over Suspension Insulator String:

A string of suspension insulators consists of a number of porcelain discs connected in series through metallic links. Fig. 9(i) shows 3-disc string of suspension insulators. The porcelain portion of each disc is in between two metal links. Therefore, each disc forms a capacitor C as shown in Fig. 9 (*ii*). This is known as *mutual capacitance* or *self-capacitance*. If there were mutual capacitance alone, then charging current would have been the same through all the discs and consequently voltage across each unit would have been the same *i.e.*, $V/3$ as shown in Fig. 9 (ii). However, in actual practice, capacitance also exists between metal fitting of each disc and tower or earth. This is known as *shunt capacitance* $C1$. Due to shunt capacitance, charging current is not the same through all the discs of the string [See Fig. 9]. Therefore, voltage across each disc will be different. Obviously, the disc nearest to the line conductor will have the maximum voltage. Thus referring to Fig. 9 (iii), $V3$ will be much more than $V2$ or $V1$.

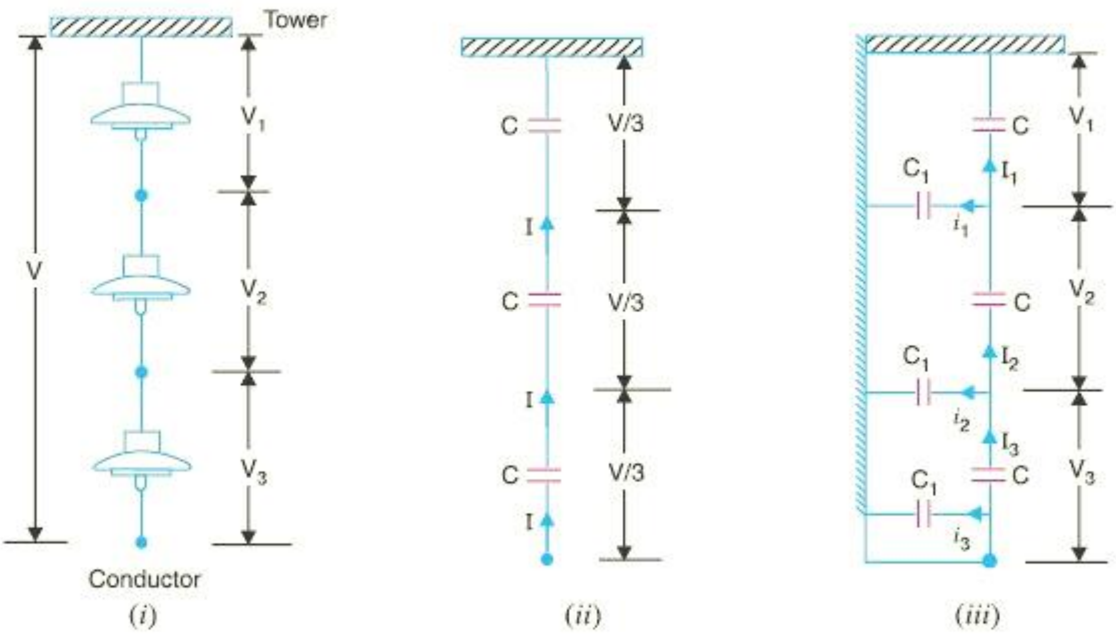


Fig. 9 Potential Distribution over Suspension Insulator String

The following points may be noted regarding the potential distribution over a string of suspension insulators:

i. The voltage impressed on a string of suspension insulators does not distribute itself uniformly across the individual discs due to the presence of shunt capacitance.

ii. The disc nearest to the conductor has maximum voltage across it. As we move towards the cross-arm, the voltage across each disc goes on decreasing.

iii. The unit nearest to the conductor is under maximum electrical stress and is likely to be punctured. Therefore, means must be provided to equalise the potential across each unit.

iv. If the voltage impressed across the string were d.c., then voltage across each unit would be the same. It is because insulator capacitances are ineffective for d.c.

String Efficiency

As stated above, the voltage applied across the string of suspension insulators is not uniformly distributed across various units or discs. The disc nearest to the conductor has much higher potential than the other discs. This unequal potential distribution is undesirable and is usually expressed in terms of string efficiency.

The ratio of voltage across the whole string to the product of number of discs and the voltage across the disc nearest to the conductor is known as **string efficiency** *i.e.*,

$$\text{String efficiency} = \frac{\text{Voltage across the string}}{\text{n} * \text{Voltage across disc nearest to conductor}}$$

Where n = number of discs in the string.

String efficiency is an important consideration since it decides the potential distribution along the string. The greater the string efficiency, the more uniform is the voltage distribution. Thus 100% string efficiency is an ideal case for which the voltage across each disc will be exactly the same. Although it is impossible to achieve 100% string efficiency, yet efforts should be made to improve it as close to this value as possible.

Mathematical expression

Fig. 10 shows the equivalent circuit for a 3-disc string. Let us suppose that self-capacitance of each disc is C. Let us further assume that shunt capacitance C_1 is some fraction K of self-capacitance *i.e.*, $C_1 = KC$. Starting from the cross-arm or tower, the voltage across each unit is V_1,V_2 and V_3 respectively as shown.

Applying Kirchhoff's current law to node A, we get,

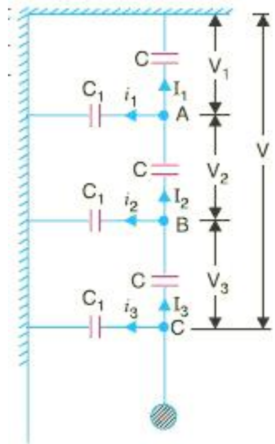


Fig. 10

$$I_2 = I_1 + i_1$$

or $$V_2 \omega C^* = V_1 \omega C + V_1 \omega C_1$$

or $$V_2 \omega C = V_1 \omega C + V_1 \omega K C$$

$$\therefore \quad V_2 = V_1 (1 + K) \quad ...(i)$$

Applying Kirchhoff's current law to node B, we get,

$$I_3 = I_2 + i_2$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2) \omega C_1 \dagger$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2) \omega K C$$

or $$\begin{aligned} V_3 &= V_2 + (V_1 + V_2)K \\ &= KV_1 + V_2 (1 + K) \\ &= KV_1 + V_1 (1 + K)^2 \\ &= V_1 [K + (1 + K)^2] \end{aligned}$$

$$\therefore \quad V_3 = V_1 [1 + 3K + K^2]$$

Voltage between conductor and earth (*i.e.,* tower) is

$$\begin{aligned} V &= V_1 + V_2 + V_3 \\ &= V_1 + V_1(1 + K) + V_1 (1 + 3K + K^2) \\ &= V_1 (3 + 4K + K^2) \end{aligned}$$

$$\therefore \quad V = V_1 (1 + K)(3 + K)$$

From expressions (*i*), (*ii*) and (*iii*), we get,

$$\frac{V_1}{1} = \frac{V_2}{1+K} = \frac{V_3}{1 + 3K + K^2} = \frac{V}{(1+K)(3+K)}$$

$$\therefore \text{Voltage across top unit,} \quad V_1 = \frac{V}{(1+K)(3+K)}$$

$$\text{Note that current through capacitor} = \frac{\text{Voltage}}{\text{Capacitive reactance}}$$

Voltage across second shunt capacitance C_1 from top = $V_1 + V_2$. It is because one point of it is connected to B and the other point to the tower.

Voltage across second unit from top, $V_2 = V_1(1 + K)$

Voltage across third unit from top, $V_3 = V_1(1 + 3K + K^2)$

$$\text{\%age String efficiency} = \frac{\text{Voltage across string}}{\text{n} * \text{Voltage across disc nearest to conductor}} * 100$$

$$= \frac{V}{3 * V_3} * 100$$

The following points may be noted from the above mathematical analysis:

1. If $K = 0{\cdot}2$ (Say), then from exp. (*iv*), we get, $V2 = 1{\cdot}2\ V1$ and $V3 = 1{\cdot}64\ V1$. This clearly shows that disc nearest to the conductor has maximum voltage across it; the voltage across other discs decreasing progressively as the cross-arm in approached.
2. The greater the value of K (= C_1/C), the more non-uniform is the potential across the discs and lesser is the string efficiency.

3. The inequality in voltage distribution increases with the increase of number of discs in the string. Therefore, shorter string has more efficiency than the larger one.

Methods of Improving String Efficiency:

It has been seen above that potential distribution in a string of suspension insulators is not uniform. The maximum voltage appears across the insulator nearest to the line conductor and decreases progressively as the cross arm is approached. If the insulation of the highest stressed insulator (*i.e.* nearest to conductor) breaks down or flash over takes place, the breakdown of other units will take place in succession. This necessitates to equalize the potential across the various units of the string *i.e.* to improve the string efficiency.

The various methods for this purpose are:

By using longer cross-arms:

The value of string efficiency depends upon the value of *K i.e.*, ratio of shunt capacitance to mutual capacitance. The lesser the value of *K*, the greater is the string efficiency and more uniform is the voltage distribution. The value of *K* can be decreased by reducing the shunt capacitance. In order to reduce shunt capacitance, the distance of conductor from tower must beincreased *i.e.*, longer cross-arms should be used as shown in fig. 11. However, limitations of cost and strength of tower do not allow the use of very long cross-arms. In practice, *K* = 0·1 is the limit that can be achieved by this method.

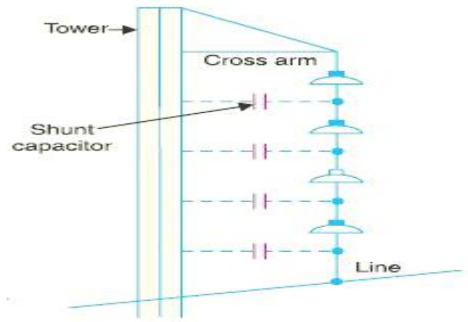


Fig. 11 longer cross-arms

By grading the insulators:

In this method, insulators of different dimensions are so chosen that each has a different capacitance. The insulators are capacitance graded *i.e.* they are assembled in the string in such a way that the top unit has the minimum capacitance, increasing progressively as the bottom unit (*i.e.*, nearest to conductor) is reached. Since voltage is inversely proportional to capacitance, this method tends to equalize the potential distribution across the units in the string. This method has the disadvantage that a large number of different-sized insulators are required. However, good results can be obtained by using standard insulators for most of the string and larger units for that near to the line conductor.

By using a guard ring:

The potential across each unit in a string can be equalized by using a guard ring which is a metal ring electrically connected to the conductor and surrounding the bottom insulator as shown in the Fig. 12. The guard ring introduces capacitance between metal fittings and the line conductor. The guard ring is contoured in such a way that shunt capacitance currents $i1$, $i2$ etc. are equal to metal fitting line capacitance currents $i'1$, $i'2$ etc. The result is that same charging current I flows through each unit of string. Consequently, there will be uniform potential distribution across the units.

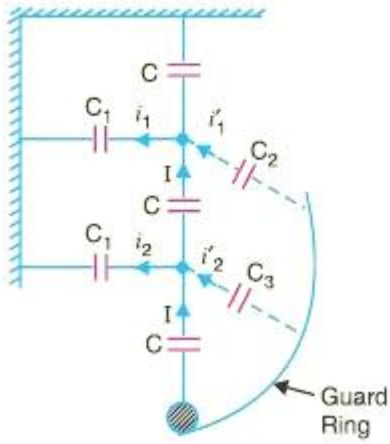


Fig. 12 Using Guard Ring

Important Points

While solving problems relating to string efficiency, the following points must be kept in mind:

The maximum voltage appears across the disc nearest to the conductor (*i.e.,* line conductor).

The voltage across the string is equal to phase voltage *i.e.,*

Voltage across string = Voltage between line and earth = Phase Voltage

$$\text{Line Voltage} = \sqrt{3} * \text{Voltage across string}$$

Corona

When an alternating potential difference is applied across two conductors whose spacing is large as compared to their diameters, there is no apparent change in the condition of atmospheric air surrounding the wires if the applied voltage is low. However, when the applied voltage exceeds a certain value, called critical disruptive voltage, the conductors are surrounded by a faint violet glow called corona.

The phenomenon of corona is accompanied by a hissing sound, production of ozone, power loss and radio interference. The higher the voltage is raised, the larger and higher the luminous envelope becomes, and greater are the sound, the power loss and the radio noise. If the applied voltage is increased to breakdown value, a flash-over will occur between the conductors due to the breakdown of air insulation.

The phenomenon of violet glow, hissing noise and production of ozone gas in an overhead transmission line is known as corona.

If the conductors are polished and smooth, the corona glow will be uniform throughout the length of the conductors, otherwise the rough points will appear brighter. With d.c. voltage, there is difference in the appearance of the two wires. The positive wire has uniform glow about it, while the negative conductor has spotty glow.

Theory of corona formation

Some ionization is always present in air due to cosmic rays, ultraviolet radiations and radioactivity. Therefore, under normal conditions, the air around the conductors contains some ionized particles (i.e., free electrons and +ve ions) and neutral molecules. When p.d. is applied between the conductors, potential gradient is set up in the air which will have maximum value at the conductor surfaces. Under the influence of potential gradient, the existing free electrons acquire greater velocities. The greater the applied voltage, the greater the potential gradient and more is the velocity of free electrons.

When the potential gradient at the conductor surface reaches about 30 kV per cm (max. value), the velocity acquired by the free electrons is sufficient to strike a neutral molecule with enough force to dislodge one or more electrons from it. This produces another ion and one or more free electrons, which is turn are accelerated until they collide with other neutral molecules, thus producing other ions. Thus, the process of ionization is cumulative. The result of this ionization is that either corona is formed or spark takes place between the conductors.

Factors Affecting Corona

The phenomenon of corona is affected by the physical state of the atmosphere as well as by the conditions of the line. The following are the factors upon which corona depends:

1. **Atmosphere:** As corona is formed due to ionization of air surrounding the conductors, therefore, it is affected by the physical state of atmosphere. In the stormy weather, the number of ions is more than normal and as such corona occurs at much less voltage as compared with fair weather.
2. **Conductor size :**The corona effect depends upon the shape and conditions of the conductors. The rough and irregular surface will give rise to more corona because unevenness of the surface decreases the value of breakdown voltage. Thus a stranded conductor has irregular surface and hence gives rise to more corona that a solid conductor.
3. **Spacing between conductors :**If the spacing between the conductors is made very large as compared to their diameters, there may not be any corona effect. It is because larger distance between conductors reduces the electro-static stresses at the conductor surface, thus avoiding corona formation.
4. **Line voltage :**The line voltage greatly affects corona. If it is low, there is no change in the condition of air surrounding the conductors and hence no corona is formed. However, if the line voltage has such a value that electrostatic stresses developed at the conductor surface make the air around the conductor conducting, then corona is formed.

The phenomenon of corona plays an important role in the design of an overhead transmission line. Therefore, it is profitable to consider the following terms much used in the analysis of corona effects:

(i) Critical disruptive voltage

It is the minimum phase-neutral voltage at which corona occurs.

Consider two conductors of radii r cm and spaced d cm apart. If V is the phase-neutral potential, then potential gradient at the conductor surface is given by:

$$g = \frac{V}{r \log \frac{d}{r}} \; volts / cm$$

In order that corona is formed, the value of g must be made equal to the breakdown strength of air. The breakdown strength of air at 76 cm pressure and temperature of 25ºC is 30 kV/cm (max) or 21·2 kV/cm (r.m.s.) and is denoted by go. If V_c is the phase-neutral potential required under these conditions, then,

$$g_o = \frac{V_c}{r \, log_e \frac{d}{r}} \; volts \, / \, cm$$

Where go = breakdown strength of air at 76 cm of mercury and 25ºC

= 30 kV/cm (max) or 21·2 kV/cm (r.m.s.)

∴ Critical disruptive voltage, $V_c = g_o \, r \, log_e \frac{d}{r}$

The above expression for disruptive voltage is under standard conditions i.e., at 76 cm of Hg and 25ºC. However, if these conditions vary, the air density also changes, thus altering the value of go. The value of go is directly proportional to air density. Thus the breakdown strength of air at a barometric pressure of b cm of mercury and temperature of tºC becomes δ go where

δ = air density factor = 3.92b/(273 + t)

Under standard conditions, the value of δ = 1.

∴ Critical disruptive voltage , $V_c = g_o \delta \, r \, log_e \frac{d}{r}$

Correction must also be made for the surface condition of the conductor. This is accounted for by multiplying the above expression by irregularity factor mo.

∴ Critical disruptive voltage, $V_c = m_o g_o \delta \, r \, log_e \frac{d}{r}$ kV/phase

where

mo = 1 for polished conductors

= 0·98 to 0·92 for dirty conductors

= 0·87 to 0·8 for stranded conductors

(ii) Visual critical voltage

It is the minimum phase-neutral voltage at which corona glow appears all along the line conductors.

It has been seen that in case of parallel conductors, the corona glow does not begin at the disruptive voltage V_c but at a higher voltage V_v , called visual critical voltage. The phase-neutral effective value of visual critical voltage is given by the following empirical formula:

$V_v = m_v g_o \delta \, r \left(1 + 0.3/\sqrt{\delta r}\right) log_e \frac{d}{r}$ kV/phase

where m_v is another irregularity factor having a value of 1·0 for polished conductors and 0·72 to 0·82 for rough conductors.

(iii) Power loss due to corona

Formation of corona is always accompanied by energy loss which is dissipated in the form of light, heat, sound and chemical action. When disruptive voltage is exceeded, the power loss due to corona is given by:

$P = 242 \cdot 2 \left(\frac{f+25}{\delta}\right) \sqrt{\frac{r}{d}} (V - V_c)^2 \times 10^{-5}$ kW / km / phase

Where f = supply frequency in Hz

V = phase-neutral voltage (r.m.s.)

V_c = disruptive voltage (r.m.s.) per phase

Advantages and Disadvantages of Corona

Corona has many advantages and disadvantages. In the correct design of a high voltage overhead line, a balance should be struck between the advantages and disadvantages.

Advantages

(*i*) Due to corona formation, the air surrounding the conductor becomes conducting and hence virtual diameter of the conductor is increased. The increased diameter reduces the electrostatic stresses between the conductors.

(*ii*) Corona reduces the effects of transients produced by surges.

Disadvantages

(*i*) Corona is accompanied by a loss of energy. This affects the transmission efficiency of the line.

(*ii*) Ozone is produced by corona and may cause corrosion of the conductor due to chemical action.

(*iii*) The current drawn by the line due to corona is non-sinusoidal and hence non-sinusoidal voltage drop occurs in the line. This may cause inductive interference with neighboring communication lines.

Methods of Reducing Corona Effect

It has been seen that intense corona effects are observed at a working voltage of 33 kV or above. Therefore, careful design should be made to avoid corona on the sub-stations or bus-bars rated for 33 kV and higher voltages otherwise highly ionized air may cause flash-over in the insulators or between the phases, causing considerable damage to the equipment. The corona effects can be reduced by the following methods :

(i) By increasing conductor size. By increasing conductor size, the voltage at which corona occurs is raised and hence corona effects are considerably reduced. This is one of the reasons that *ACSR* conductors which have a larger cross-sectional area are used in transmission lines.

(ii) By increasing conductor spacing. By increasing the spacing between conductors, the voltage at which corona occurs is raised and hence corona effects can be eliminated. However, spacing cannot be increased too much otherwise the cost of supporting structure (*e.g.,* bigger cross arms and supports) may increase to a considerable extent.

Radio Interference

Radio interference is the adverse effect introduced by corona on wireless broadcasting. The corona discharge emit radiation which may introduce noise signals in the communication lines, radio and television receivers. It is mainly due to brush discharge on the surface irregularities of the conductor during positive half cycles. This leads to corona loss at voltages lower than the critical voltages.

The negative discharge are less troublesome for radio reception. Radio interference is considered as a field measured in microvolt per meter at any distance from the transmission line and is significant only at voltages greater than 200 kv. There is gradual increase in RI level till the voltage is such that measurable corona loss takes place. Above this voltage there is rapid increase in RI level. The rate of increase is more for smooth and large diameter conductors. The amplitude of RI level varies inversely as the frequency at which the interference is measured. Thus the services in the higher frequency band e.g., television, frequency modulated broadcasting, microwave relay, radar etc. are less affected. Radio interference is one of the very important factors while designing a transmission line.

Assignment-Cum-Tutorial Questions

A. *Objective Questions*

1. Higher the frequency, _____________.
   a. Lower the corona loss. **b.** Higher is the corona loss.
   c. Does not effect. d. Depends on the physical conditions.

2. What is the use of bundled conductors?
   a. Reduces surface electric stress of conductor. b. Increases the line reactance.
   c. Decreases the line capacitance. d. None of these.

3. The corona loss in a transmission line depend on?
   a. Material of the conductor. **b.** Diameter of the conductor.
   c. Height of the conductor. d. None of these.

4. What is corona?
   a. Partial breakdown of air. **b.** Complete breakdown of air.
   c. Sparking between the lines. d. None of these.

5. In which climate does the chances of occurrence of corona is maximum?
   a. Dry b. Hot summer. c. Winter. **d.** Humid.

6. Which gas is formed near to the conductors by producing a hissing noise?
   a. Oxygen. **b.** Ozone. c. Hydrogen. d. Nitrogen.

7. Corona is observed on
   a. AC transmissions only. b. DC transmissions only.
   c. AC and DC transmissions. d. None of these.

8. Which harmonics are generated during the corona, which leads to the increase in corona losses?
   a. Third harmonics. b. Fifth harmonics. c. Seventh harmonics. d. None of these.

9. Corona loss can be reduced by using
   a. Solid conductor. b. Hollow conductor. c. Bundle conductor. **d**. both c & b .

10. What is the effect on corona, if the spacing between the conductors is increased?
    a. Corona increases**.** **b.** Corona is absent. c. Corona decreases. d. None of these.

11. Why are the hollow conductors used?

a. Reduce the weight of copper. b. Improve stability.

c. Reduce corona d. Increase power transmission capacity.

12. Which of these given statements is wrong in consideration with bundled conductors?

a. Control of voltage gradient. b. Reduction in corona loss.

c. Reduction in the radio interference. d. Increase in interference with communication lines.

13. Why are bundled conductors employed?

a. Appearance of the transmission line is improved.

b. Mechanical stability of the line is improved.

c. Improves current carrying capacity.

d. Improves the corona performance of the line.

14. The effect of dirt on the surface of the conductor is to ______________ irregularity and thereby _________________ the break down voltage.

a. Decreases, reduces. b. Increases, increases. c. Increases, reduces. d.none

15. Find the spacing between the conductors a 132 kV 3 phase line with 1.956 cm diameter conductors is built so that corona takes place, if the line voltage exceeds 210 kV (rms). With go = 30 kV/cm.

a. 1.213 m b. 2.315 m. c. 3.451 m. d. 4.256 m.

16. Air density factor which depends on the pressure and temperature is given by which of these formulas?

a. $\delta = (5.92 * b) / (273 + t)$ b. $\delta = (7.92 * b) / (273 + t)$

c. $\delta = (3.92 * b) / (273 + t)$ d. $\delta = (4.92 * b) / (273 + t)$.

17. When the voltage applied is equal to the critical disruptive voltage what happens?

a. Corona does not start. b. Corona starts but is not visible.

c. Corona starts and is visible. d. Only ionization starts.

18. What is the critical disruptive voltage of a three phase line which has a conductor of 2 cm in diameter which are spaced equilaterally 1m apart, if the dielectric strength of air is about 30 kV/cm, and air density about 0.952 and irregularity factor 0.9?

a. 152.36 kV b. 146.235 kV c. 144.95 kV d. 111.235 kV

19. In case of dc voltage, which colour beads are formed near the negative conductor?

a. Reddish. b. Bluish. c. Greenish. d. Violet.

20. In case of dc voltage, smoother _____________ uniform glow near the positive conductor.

    a. Yellow. b. Bluish white. c. Reddish white. d. Greenish yellow.

21. In a string of suspension insulators, the voltage distribution across the different units of a string could be made uniform by the use of a grading ring, because it [ ]

    a) Forms capacitances with link-pins to carry the charging current from link pins

    b)Forms capacitances which help to cancel the charging current from link pins

    c) Increases the capacitances of lower insulator units of cause equal voltage drop

    d) Decreases the capacitances of upper insulators units to cause equal voltage drop

22. What is the sag for a span of 400m, if the ultimate tensile strength of conductor is 6000kgf, and the weight of conductor is 550 kgf/km? Factor of safety is 2. [ ]

    a) 1.016 m b) 2.40 m **c)** 3.6 m d) 4.2 m

23. The non-uniform distribution of voltage across the units in a string of suspension type insulators is due to [ ]

    a) Unequal self-capacitance of the units

    b) Non-uniform distance of separation of the units from the tower body

    c)The existence of stray capacitance between the metallic junctions of the units andthe tower body

    d) Non-uniform distance between the cross-arm and the units

24. Pin type insulators are used up to _______ voltages. [ ]

    a) 220V b) 400V c) 11KV **d)**33KV

25. Whenever the conductors are dead-ended or there is a change in the direction of transmission line, the insulators used are of the [ ]

    a) Pin type b) suspension type **c)** strain type d) shackle type

26. The insulation of modern EHV lines is designed based on [ ]

    a) The lightning voltage b) corona c) radio interface **d)** switching voltage

27. Electrical conductivity of insulators is the range _____________.

    a) $10^{-10}(\Omega\text{-mm})^{-1}$ b) $10^{-10}(\Omega\text{-cm})^{-1}$ c) $10^{-10}(\Omega\text{-m})^{-1}$ d) $10^{-8}(\Omega\text{-m})^{-1}$

28. In a string of suspension insulators, the voltage across the line unit is [ ]

    a) Maximum b) minimum c) zero d) equal to line voltage

B. Descriptive Questions

1. What is corona? What are the factors which affect corona?

2. Explain the phenomenon of corona? How can the corona loss be minimized in transmission lines.
3. How a corona formation does affect the efficiency of the line? Give Peterson’s formula to determine the power loss due to corona?
4. Describe the various methods for reducing corona effect in an overhead transmission line.
5. What is critical disruptive voltage? Derive the expression for this voltage?
6. Explain the following terms with reference to corona:
7. i) Critical disruptive voltage
8. ii) Visual critical voltage
9. iii) Power loss due to corona
10. What is strain insulator and where is it used? Give a sketch to show its location.
11. Explain various methods of improving string efficiency.
12. Why String efficiency for a d.c system is 100%? Discuss.
13. List the characteristics which the insulator should possess.
14. What do you mean by string efficiency? How can it be improved?
15. What are different types of insulators used in transmission and distribution systems and explain them?
16. How do elasticity and temperature affect the sag and tension of the conductor? Find out the expression of tension when both of them are taken into account.
17. Deduce an approximate expression for sag in overhead lines when (i) supports are at equal levels (ii) supports are at unequal levels.
18. Explain effect of wind and ice on sag of the conductor.
19. Explain the necessity of a stringing chart for a transmission line and show how such a chart can be constructed.

20. A 3-phase line has conductors 2 cm in diameter spaced equilaterally 1 m apart.If the dielectric strength of air is 30 kV (max) per cm, find the disruptive critical voltage for the line.Take air density factor $\delta = 0{\cdot}952$ and irregularity factor mo = 0·9.[Ans: 144·8 kV]

21. Taking the dielectric strength of air to be 30 kV/cm, calculate the disruptive critical voltage for a 3-phase line with conductors of 1 cm radius and spaced symmetrically 4 m apart. [Ans:220 kV line voltage]

22. A 3-phase, 220 kV, 50 Hz transmission line consists of 1·2 cm radius conductors spaced 2 m at the corners of an equilateral triangle. Calculate the corona loss per km of the line. The condition of the wire is smoothly weathered and the weather is fair with temperature of 20ºC and barometric pressure of 72·2cm of Hg. [Ans: 2·148 kW]

23. A 3-phase, 220 kV, 50 Hz transmission line consists of 1·5 cm radius conductor spaced 2 meters apart in equilateral triangular formation. If the temperature is 40ºC and atmospheric pressure is 76 cm, calculate the corona loss per km of the line. Take mo = 0·85.[Ans :0·05998 kW]

24. A 132 kV line with 1·956 cm dia. conductors is built so that corona takes place if the line voltage exceeds 210 kV (r.m.s.). If the value of potential gradient at which ionization occurs can be taken as 30 kV per cm, find the spacing between the conductors. .[Ans :341cm]

25. Find the critical disruptive voltage and visual critical voltages for local and general corona on a 3phase transmission line,consisting of 3 stranded copper conductors spaced 2.5m apart at the corners of an equilateral triangle.air temperature and pressure are 21^0C and 73.6 cm of Hg respectively. The conductor diameter ,irregularity factor and surface factors for local and general corona are 10.4mm,0.85,0.72 and 0.82 Respectively.[Ans **:98.36kv, vv =118.43kv , general v_v=150.15kv]**

26. A 3-phase, 230 kV, 50 Hz transmission line consists of 1•5 cm radius conductor spaced 2.5 meters apart in equilateral triangular formation. If the temperature is 45ºC and atmospheric pressure is 76 cm, calculate the corona loss per km of the line. Take mo = 0•85.[**Ans :P_{3phase} =0•469 kW/km]**

27. A string of 5 insulators is connected across a100kV line. If the capacitance of each disc to earth is 0.1 of the capacitance of the insulator, calculate (i) the distribution of voltage on the insulator discs and (ii) the string efficiency.
    Ans:$V_{1=}$ 7.96KV, $V_{2=}$8.77KV, $V_{3=}$10.5KV,$V_{4=}$13.16KV, $V_{5=}$17.3KV, String Efficiency=66.7%

28. An insulator string consists of three units, each having a safe working voltage of 15kV. The ratio of self-capacitance to shunt capacitance of each unit is 8:1. Find the maximum safe working voltage of the string. Also find the string efficiency.
    Ans: V=37.92KV, String Efficiency=84.26%

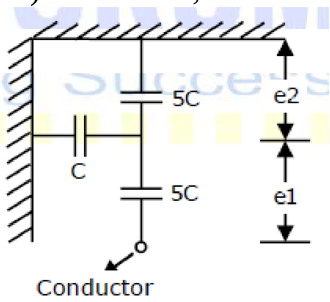
29. A string of suspension insulator consists of three units.the capacitance between each link pin and earth is $1/8^{th}$ of self-capacitance of the unit. if the maximum peak voltage per unit is not to exceed 20kv, find the maximum voltage that can be applied across the string and string efficiency.
    Ans: V=35.741KV, String Efficiency=84.3%

30. In a 33kV overhead line, there are three units in the string of insulators. If the capacitance between each insulator pin and earth is 11% of self-capacitance of each insulator, find (i) the distribution of voltage over 3 insulators and (ii) string efficiency.
    Ans:$V_{1=}$ 5.518KV, $V_{2=}$ 6.13KV, $V_{3=}$ 7.406KV, String Efficiency=85.8%

C. Gate/ IES Questions

1. Consider a three-phase, 50Hz, 11kV distribution system. Each of the conductors issuspended by an insulator string having two identical porcelain insulators. The self-capacitance of the insulator is 5 times the shunt capacitance between the link andthe ground, as shown in the figure. The voltage across the two insulators is [ ]
   a) e1 = 3.74kV, e2 = 2.61kV **b)** e1 = 3.46kV, e2 = 2.89kV

c) e1 = 6.0kV, e2 = 4.23kV d) e1 = 5.5kV, e2 = 5.5kV



GATE-10

2. The insulation strength of an EHV transmission line is mainly governed by [ ]
   a) load power factor **b)** switching over-voltages
   c) harmonics d) corona **GATE-05**

3. The insulation level of a 400 KV EHV overhead transmission line is decided on thebasis of
   [ ]
   a) Lightning over voltage **b)** switching over voltage
   c) Corona inception voltage d) radio and TV interference **GATE-12**

4. Bundled conductors are mainly used in high voltage overhead transmission lines to **GATE2003**
   a. reduce transmission line losses b. increase mechanical strength of the line
   c .reduce corona d. reduce sag

5. The corona loss on a particular system at 50 Hz is 1 KW/km per phase. The corona loss at 60 Hz would be **GATE 2000**
   a. 1 KW/km per phase . b. 0.83 KW/km per phase.
   c. 1.2 KW/km per phase. d. 1.13 KW/km per phase.

6. Corona losses are minimized when **GATE1999**
   a. conductor size is reduced b. smooth conductor is reduced
   c.sharp ponts are provided in the line hardware d.current density in conductors is reduced.

UNIT-VI

Sag and Tension Calculations

Sag in Overhead Lines

While erecting an overhead line, it is very important that conductors are under safe tension. If the conductors are too much stretched between supports in a bid to save conductor material, the stress in the conductor may reach unsafe value and in certain cases the conductor may break due to excessive tension. In order to permit safe tension in the conductors, they are not fully stretched but are allowed to have a dip or sag.

The difference in level between points of supports and the lowest point on the conductor is called **sag.**

Fig. 1 Shows a conductor suspended between two equilevel supports *A* and *B*. The conductor is not fully stretched but is allowed to have a dip. The lowest point on the conductor is *O* and the sag is *S*. The following points may be noted:

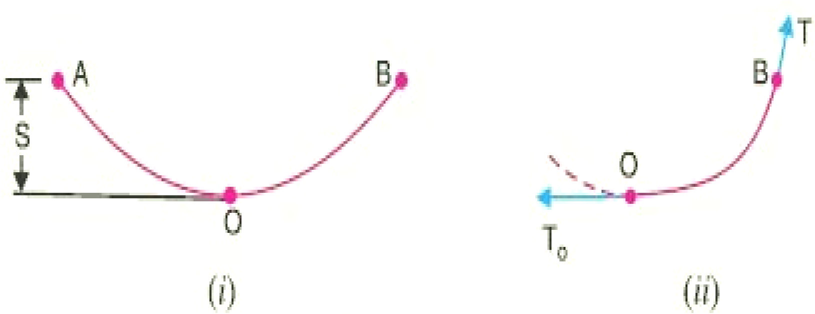


Fig. 1

i. When the conductor is suspended between two supports at the same level, it takes the shape of catenary. However, if the sag is very small compared with the span, then sag-span curve is like a parabola.
ii. The tension at any point on the conductor acts tangentially. Thus tension *TO* at the lowest point *O* acts horizontally as shown in Fig. 8.23. (*ii*).
iii. The horizontal component of tension is constant throughout the length of the wire.

iv. The tension at supports is approximately equal to the horizontal tension acting at any point on the wire. Thus if *T* is the tension at the support *B*, then *T* = *TO*.

Conductor sag and tension:

This is an important consideration in the mechanical design of overhead lines. The conductor sag should be kept to a minimum in order to reduce the conductor material required and to avoid extra pole height for sufficient clearance above ground level. It is also desirable that tension in the conductor should be low to avoid the mechanical failure of conductor and to permit the use of less strong supports. However, low conductor tension and minimum sag are not possible. It is because low sag means a tight wire and high tension, whereas a low tension means a loose wire and increased sag. Therefore, in actual practice, a compromise in made between the two.

Calculation of Sag

In an overhead line, the sag should be so adjusted that tension in the conductors is within safe limits. The tension is governed by conductor weight, effects of wind, ice loading and temperature variations. It is a standard practice to keep conductor tension less than 50% of its ultimate tensile strength *i.e.*, minimum factor of safety in respect of conductor tension should be 2. We shall now calculate sag and tension of a conductor when (*i*) supports are at equal levels and (*ii*) supports are at unequal levels.

When supports are at equal levels:

Consider a conductor between two equilevel supports *A* and *B* with *O* as the lowest point as shown in Fig. 2. It can be proved that lowest point will be at the mid-span.

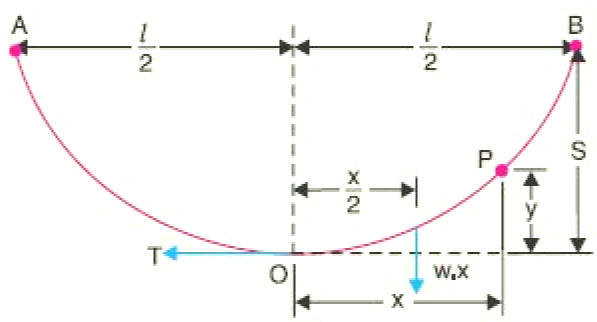


Fig. 2

Let

l = Length of span

w = Weight per unit length of conductor

T = Tension in the conductor.

Consider a point P on the conductor. Taking the lowest point O as the origin, let the co-ordinates of point P be x and y. Assuming that the curvature is so small that curved length is equal to its horizontal projection (*i.e.*, $OP = x$), the two forces acting on the portion OP of the conductor are:

i. The weight $wx$of conductor acting at a distance $x/2$ from O.

ii. The tension T acting at O

Equating the moments of above two forces about point O, we get,

$$Ty = wx * \frac{x}{2}$$

$$y = \frac{wx^2}{2T}$$

The maximum dip (sag) is represented by the value of y at either of the supports A and B.

At support A,

$$x = \frac{l}{2} \quad and \quad y = S$$

$$\therefore \qquad Sag. \quad S = \frac{w\left(\frac{l}{2}\right)^2}{2T} = \frac{wl^2}{8T}$$

When supports are at unequal levels:

In hilly areas, we generally come across conductors suspended between supports at unequal levels. Fig. 3 shows a conductor suspended between two supports A and B which are at different levels. The lowest point on the conductor is O.

Let

l = Span length

h = Difference in levels between two supports

X_1 = Distance of support at lower level (*i.e.*, A) from O

X_2 = Distance of support at higher level (*i.e.* B) from O

T = Tension in the conductor

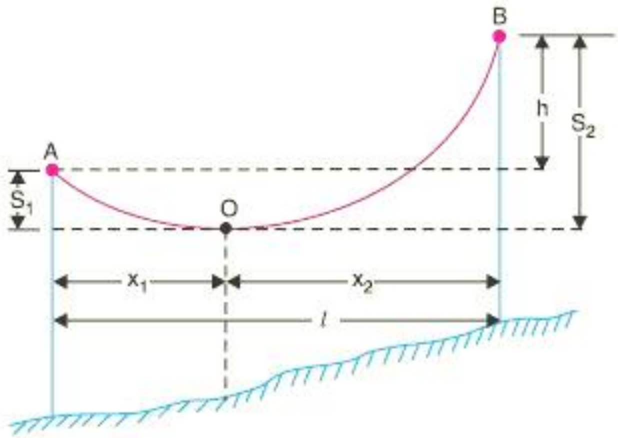


Fig. 3

If w is the weight per unit length of the conductor, then,

$$Sag\ S_1 = \frac{WX_1^2}{2T}$$

And

$$Sag\ S_2 = \frac{WX_2^2}{2T}$$

$$x_1 + x_2 = l$$

Now $\quad S_2 - S_1 = \frac{w}{2T}[x_2^2 - x_1^2] = \frac{w}{2T}(x_2 + x_1)(x_2 - x_1)$

$\therefore \quad S_2 - S_1 = \frac{w\,l}{2T}(x_2 - x_1) \qquad [\because x_1 + x_2 = l]$

But $\quad S_2 - S_1 = h$

$\therefore \quad h = \frac{w\,l}{2T}(x_2 - x_1)$

or $\quad x_2 - x_1 = \frac{2\,Th}{w\,l} \qquad ...(ii)$

Solving exps. (*i*) and (*ii*), we get,

$$x_1 = \frac{l}{2} - \frac{T\,h}{w\,l}$$

$$x_2 = \frac{l}{2} + \frac{T\,h}{w\,l}$$

Having found X_1 and X_2 values of S1 and S2 can be easily calculated.

Effect of wind and ice loading:

The above formulae for sag are true only in still air and at normal temperature when the conductor is acted by its weight only. However, in actual practice, a conductor may have ice coating and simultaneously subjected to wind pressure. The weight of ice acts vertically downwards *i.e.*, in the same direction as the weight of conductor. The force due to the wind is assumed to act horizontally *i.e.*, at right angle to the projected surface of the conductor.
Hence, the total force on the conductor is the vector sum of horizontal and vertical forces as shown in Fig. 4

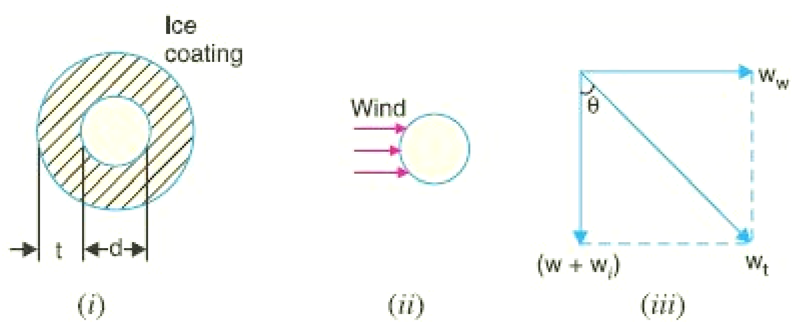


Fig. 4

Total weight of conductor per unit length is

$$w_t = \sqrt{(w + w_i)^2 + (w_w)^2}$$

where w = weight of conductor per unit length

= conductor material density × volume per unit length

w_i = weight of ice per unit length

= density of ice × volume of ice per unit length

$= \text{density of ice} \times \frac{\pi}{4}[(d+2t)^2 - d^2] \times 1$

$= \text{density of ice} \times \pi\, t\,(d+t)$*

w_w = wind force per unit length

= wind pressure per unit area × projected area per unit length

$= \text{wind pressure} \times [(d + 2t) \times 1]$

When the conductor has wind and ice loading also, the following points may be noted:

The conductor sets itself in a plane at an angle θ to the vertical where

$$\tan\theta = \frac{W_w}{W + W_i}$$

The sag in the conductor is given by:

$$S = \frac{W_t l^2}{2T}$$

Hence S represents the slant sag in a direction making an angle θ to the vertical. *If no specific mention is made in the problem, then slant slag is calculated by using the above formula.*

The vertical sag $S = S\cos\theta$

Working stress= $\frac{\text{Ultimate Strength}}{\text{Safety factor}}$

Working Tension, T= Working stress * conductor area

Assignment-Cum-Tutorial Questions

A. *Objective Questions*

1. What is the sag for a span of 400m, if the ultimate tensile strength of conductor is 6000kgf, and the weight of conductor is 550 kgf/km? Factor of safety is 2. [ ]
   a) 1.016 m b) 2.40 m **c)** 3.6 m d) 4.2 m

2. Between two equal supports, due to sag the conductor takes the form of [ ]
   a) Semi-circle **b)** Catenary c) Hyperbola d) None

3. The effect of ice deposition on conductor is__________ [ ]
   a)Increased sag b) Reduced sag
   c) Increased skin effect d) reduced corona loass

4. The sag of a transmission line is least affected by [ ]
   a) Current through conductor b) self-weight of conductor
   c) Ice deposited on conductor d) temperature of surrounding air.

5. In a transmission line sag depends on [ ]
   a) Conductor material b) tension in conductor
   c) Weight of conductor per unit length **d)** all above

6. The effect of ice loading on transmission line conductors is to increase [ ]
   a) Tendency of corona b) transmission losses
   c)sag of the conductor d) resistance to flow of current

7. Galloping of conductors refers to [ ]
   a)standing b) temperature change **c)**vibration d) spacing and transposition

8. The purpose of Guard ring is [ ]
   a) To reduce transmission losses **b)** To reduce earth capacitance of lowest unit
   c) To increase earth capacitance of lowest unit d) None

II) Descriptive Questions

1. How do elasticity and temperature affect the sag and tension of the conductor? Find out the expression of tension when both of them are taken into account.
2. Deduce an approximate expression for sag in overhead lines when (i) supports are at equal levels (ii) supports are at unequal levels.

3. Explain effect of wind and ice on sag of the conductor.
4. Explain the necessity of a stringing chart for a transmission line and show how such a chart can be constructed.
5. Two towers of height 30 and 90 meters respectively support a transmission line conductor at a water crossing. The horizontal distance between the towers is 500 m. If the allowable tension in the conductor is 1600kg, find the minimum clearance of the conductor and the clearance of the conductor mid-way between the supports. Weight of the conductor is 1.5kg/m. Bases of the towers can be considered to be at the water level.
   Ans: Clearance = 30.7m

6. A 132 kV transmission line has the following data:

   Weight of conductor = 680 kg/km

   Length of span = 260 m

   Ultimate strength = 3100 kg.

   Safety factor =2

   Calculate the height above ground (in meters) at which the conductor should be supported. Ground clearance required is 10 meters.

7. A transmission line has a span of 150 m between level supports. The conductor has a cross-sectional area of 2cm^2. The tension in the conductor is 2000kg. If the specific gravity of the conductor material is 9.9 g/cm3 and wind pressure is 1.5 kg/m length, calculate the sag. What is the vertical sag?
   Ans: Sag= 3.493m, Vertical Sag= 2.784m.

8. A transmission line conductor at a river crossing is supported from two towers at heights of 50 and 80 meters above water level. The horizontal distance between the towers is 300m. If the tension in the conductor is 2000kg, find the clearance between the conductor and water at a point midway between the towers. Weight of conductor is 0.844 kg. Assume that the conductor takes the shape of a parabola.
   Ans: Clearance = 60.252m

9. Two towers of height 30 and 90 meters respectively support a transmission line conductor at a water crossing. The horizontal distance between the towers is 500 m. If the allowable tension in the conductor is 1600kg, find the minimum clearance of the conductor and the clearance of the conductor mid-way between the supports. Weight of the conductor is 1.5kg/m. Bases of the towers can be considered to be at the water level.
   Ans: Clearance = 30.7m

10. A transmission line conductor having a diameter of 19.5 mm, weight 850 kglkm.The span is 275 m. The wind pressure is 39 kg/m of projected area with ice coating of13 mm. The

ultimate strength of the conductor is'8000 kg. What is the maximum sag,if the factor of safety is 2 and ice weight 910 kg/m3?
Ans: Maximum Sag= 6.4m

11. A transmission line conductor at a river crossing is supported from two towers at height of 30 m and 90m, above water level. The horizontal distance between the towers is 270m, if the tension in the conductor is 1800 kg and the conductor weight 1kg/m. What is the clearance between the conductor and the water at a point midway between the towers?
Ans: 54.94m

12. An overhead transmission line has a span of 240m between level supports. What is the maximum sag if the conductor weight 727 kg/km and has a breaking strength of 6880 kg? Allow the factor of safety of 2. Neglecting wind and ice loading.
Ans: 1.52m

13. The line conductor of a transmission line has an overall diameter of 19.53 mm, weight 0.844 kg/m and an ultimate breaking strength of 7950 kg. If the factor of safety is to be 2, when conductor has an ice of 1 kg/m and a horizontal wind pressure of 1.5kg/m. What is the vertical sag, corresponding to this loading for a 300 m span level supports?
Ans: 5.22m

C. Gate/ IES Questions

Nil